Accumulated Extraction

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Let the relative extent of a particular energy extraction process be (0<p<1). By repeated application of energy extraction units, k-times, the accumulated extraction is:

$$Æ(p,k) = p + p\cdot(1-p) + p\cdot(1-p-p\cdot(1-p)) + p\cdot(1-p-p\cdot(1-p) - p\cdot(1-p-p\cdot(1-p))) + \cdots$$

Not afraid of the increasing complexity we evaluate the difference between successive terms:

$$Æ(p,k) = Æ(p,k-1) + p\cdot(1-Æ(p,k-1)) = p + (1-p)\cdot Æ(p,k-1)$$

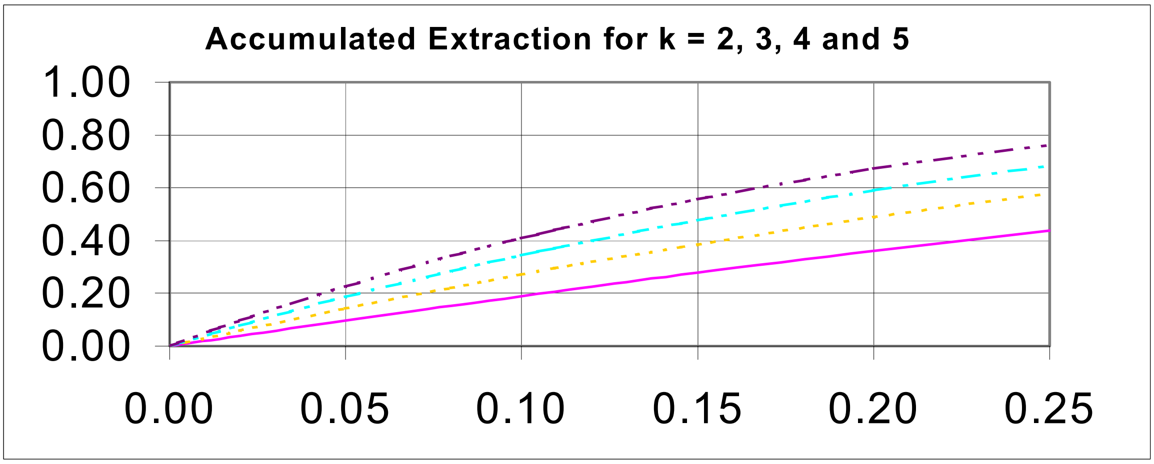
Starting with Æ(p,1) = p, we evaluate successive terms and finally get:

$$Æ(p,k) = k\cdot p - \frac{k\cdot(k-1)\cdot p^2}{2} + \frac{k\cdot(k-1)\cdot(k-2)\cdot p^3}{6} - \cdots = \sum_{i=1}^{k}\binom{k}{i}\cdot(-1)^{i-1}\cdot p^i$$

Here we recognize the coefficients in the Binominal Expansion. We also write the formula for p and k:

$$Æ(p,k) = 1-(1-p)^k \qquad p = 1-\sqrt[k]{1-Æ(p,k)} \qquad k = \frac{\log(1-Æ(p,k))}{\log(1-p)}$$

Graph I



Example:
4 units, each extracting 10%, give the accumulated extraction as Æ(0.1,4) = 0.3439 =34.39%:

$$100\% \xrightarrow[10\%]{} 90\% \xrightarrow[9\%]{} 81\% \xrightarrow[8.1\%]{} 72.9\% \xrightarrow[7.29\%]{} 65.61\%$$