# Optimizing Electrical Connections for Hot and Cold Terminals.

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## Description of the problem:

When making electrical connections to hot or cold terminals, we have to live with a temperature gradient along the connecting wire. To minimise this powerloss, we can keep the connector both long and thin, up to a point of loosing energy as resistive Joul-heat. To first order, let the thermal gradient be constant along a conductor of length (l) and cross-sectional area (A). If (Kappa) is the thermal conductivity, (R) is the total resistance of the conductor and (i) the current, the following is the power loss to 1st order if no significant Joule heating occurs:

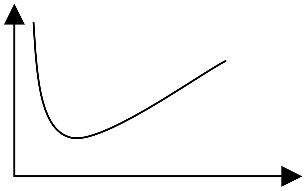
$$P_{Loss} = R \cdot i^2 + A \cdot \kappa \cdot \frac{\Delta T}{l} = \frac{i^2}{\sigma} \cdot \left(\frac{l}{A}\right) + \kappa \cdot \Delta T \cdot \left(\frac{A}{l}\right)$$

Take the conductor-length-derivative of this power and equate the result to zero. It is then easy to prove that a minimum condition exists. The graph on the right illustrates the power (P) as a function of length (l):

$$\frac{\partial P_{Loss}}{\partial l} = \frac{i^2}{\sigma} \cdot \left(\frac{1}{A}\right) - \kappa \cdot \Delta T \cdot \left(\frac{A}{l^2}\right) = 0$$

$$\Rightarrow \qquad \frac{A}{l} = \frac{i}{\sqrt{\sigma \cdot \kappa \cdot \Delta T}} = \frac{i}{H_0}$$

$$Define: \qquad H_0 = \sqrt{\sigma \cdot \kappa \cdot \Delta T}$$



Observe a new magnetic field ($H_0$). It depends on the particular conductor used and the temperature difference in question. Now insert the minimal geometry ratio (A/l) into the powerloss equation and get:

$$P_0 = 2 \cdot i \cdot \sqrt{\frac{\kappa \cdot \Delta T}{\sigma}} = i \cdot \frac{2 \cdot H_0}{\sigma} = i \cdot u_0$$

$$Define: \qquad u_0 = 2 \cdot \sqrt{\frac{\kappa \cdot \Delta T}{\sigma}} = 2 \cdot \frac{H_0}{\sigma}$$

Observe a new electric potential, ($u_0$) which allow us to compare different conductors. Let us look at the four best conductors and calculate $u_0$ and $H_0$ for a 100°C temperature difference:

| ***Metal*** | **$u_0$** (millivolts) | **$H_0$** (megaamperes/meter) |
|---|---|---|
| Gold | 34 | 1.15 |
| Silver | 39 | 1.62 |
| Copper | 51 | 1.52 |
| Aluminium | 98 | 0.88 |

Gold is the best choice, being a better electric than thermal conductor. Now solve for the length of a conductor if a current of magnitude 10 Amperes, is flowing in a 1mm diameter Gold conductor:

$$l = \frac{A \cdot H_0}{i} = \frac{\pi \cdot r^2 \cdot H_0}{i} = \frac{3.14 \cdot 25 \cdot 10^{-8} \cdot 1.15 \cdot 10^{6}}{10} = 90mm$$

Silver gives 127mm, Copper 119mm but Aluminium 69mm. The power loss is smallest for Gold being 0.34 Watts for the 10 Ampere current.