Relativistic Thermoelectromagnetism, ICT2003-P1-B4-8.

Guðlaugur Kristinn Óttarsson, Mr.,
Pro%Nil Systems, Senior Research,
Flókagötu 27, 105 Reykjavik, Iceland,
gko@islandia.is, Tel: +354-5510195.

1. Abstract

A new Vector-Scalar notation, inspired from Maxwell's Electromagnetic Equations, and a new thermoelectric speed-constant, attributed to any material or medium, is jointly used to derive a fully relativistic theory about the energy, momentum and charge transport in the presence of thermal gradients. Thermomagnetic effects are inherent with more rigor than from a phenomenological treatment alone. By reconsidering the thermoelectric transport parameters and eliminating duplicate definitions, we are in a better position to obtain analytical solutions to nonlinear and dynamic thermoelectromagnetic problems.

2.1 Introduction to *Photor* Notation

The minimal algebra of a vector-scalar hybrid we name *Photor* discovered or unfolded from Maxwell's Electromagnetic Theory is introduced. We start with the Light-cone-photor "s=r+ict". Composition, conjugation and decomposition rules are identical to the rules for the classical complex plane: Eq 2.1.1a-d

$$\tilde{s} = \vec{r} + ict \qquad \vec{r} = \tfrac{1}{2}(\tilde{s} + \tilde{s}^*)$$
$$\tilde{s}^* = \vec{r} - ict \qquad ict = \tfrac{1}{2}(\tilde{s} - \tilde{s}^*)$$

Two scalar products are possible, the internal sum of products and the internal sum of conjugate products, both a real number, the latter positive definite, and a natural choice for the squared Photor-Norm (s^{*2}): Eq 2.1.2a-b

$$\tilde{s} \circ \tilde{s} = \vec{r}^{\,2} - c^2 t^2 \qquad \tilde{s} \circ \tilde{s}^* = \vec{r}^{\,2} + c^2 t^2$$

Observe the Lorentz-Einstein equation for propagation of light signals (s^2) and the positive norm (s^{*2}). Readers familiar with "Quaternions" will appreciate Photors!

2.2 Cross Products and Thor Products for *Photors*

The set of elements in a Photor Product is the linear composition, of all possible pairs, formed in combining all elements of two *Photors*. To exhaust sign permutations, two will be needed. Name the two operations *Cross Product* (×) and *Thor Product* (⊗): Eq 2.2.1a-b

$$\tilde{s}\times = \begin{bmatrix} ct & -z & y & ix \\ z & ct & -x & iy \\ -y & x & ct & iz \\ -ix & -iy & -iz & ct \end{bmatrix}, \quad \tilde{s}\otimes = \begin{bmatrix} ct & -z & y & ix \\ z & ct & -x & iy \\ -y & x & ct & iz \\ ix & iy & iz & ct \end{bmatrix}$$

It is easy to verify that "***s x s = -is²*** " is a pure scalar. A Cross Product or a Thor Product will be named "Cross-curl" or "Crossed-curl" when the operator is the Photor-Derivative, "$\tilde{\nabla} = \vec{\nabla} - (i/c)\cdot(\partial/\partial t)$" as seen in Eq 2.2.2.a-b

$$\tilde{\nabla} \times \tilde{A} = \vec{\nabla} \times \vec{A} + \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{A} + \tfrac{1}{c}\cdot\vec{\nabla}\phi + i\cdot\left(\vec{\nabla}\circ\vec{A} + \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$
$$\tilde{\nabla} \otimes \tilde{A} = \vec{\nabla} \times \vec{A} + \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{A} + \tfrac{1}{c}\cdot\vec{\nabla}\phi - i\cdot\left(\vec{\nabla}\circ\vec{A} - \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$

Magnetic Flux Density, the negative Electric Field and the Gauge appear here as a collective entity in both curls.

2.3 Energy & Momentum in *Photor* Notation

The vector potential (**A**) is not a potential in the electric sense, but rather a momentum per charge as revealed in the expression "**p**=mv+q**A**". The canonical momentum (**p**), when extended to *Photor*, becomes "**p**+***i***cm". The scalar part consists of the Light Speed (c) times the mass (m). This momentum becomes the Einstein energy mc^2 when multiplied with the Light Speed: Eq 2.4.1a-d

$$\tilde{p} = \vec{p} + i\cdot c\cdot m, \qquad \tilde{A} = \vec{A} + i\cdot\phi/c$$
$$\tilde{U} = c\cdot\tilde{p} = c\cdot\vec{p} + i\cdot c^2\cdot m, \qquad \tilde{\Phi} = c\cdot\tilde{A} = c\cdot\vec{A} + i\cdot\phi$$

The positive definite norm of the *Energy Photor U* is equal to the relativistic energy for a mass m. A lack of name compels us to use "Momentum per Charge" for the Photor (***Ã***). Voltage on the other hand is recognized as energy per charge. It is now obvious that the vector (c**A**) qualifies to bear the name "Vector Potential". The new *Voltage Potential Photor* (***Φ***) multiplied with the *Current Density* Photor (**j**), gives Energy-flux: Eq 2.4.2a-b

$$\text{Current Density:} \quad \tilde{j} = \vec{j} + i\cdot c\cdot\rho$$
$$\text{Power Density:} \quad \tilde{\Phi}\times\tilde{j} = c\cdot\tilde{A}\times\tilde{j}$$
$$= c\cdot\vec{A}\times\vec{j} + \phi\cdot\vec{j} + c^2\cdot\rho\cdot\vec{A} + i\cdot c\cdot\left(\vec{A}\circ\vec{j} + \rho\cdot\phi\right)$$

The scalar part in the expression for Electromagnetic Power Density must be identified with *pressure*. That can be seen when compared with a particle-flux and energy in mechanics. To do so, we employ identical particles of mass (m) and density (n) and particle flux (***J***): Eq 2.4.3a-b

$$\text{Matter Flux:} \quad \tilde{J} = \vec{J} + i\cdot c\cdot n$$
$$\text{Power Density:} \quad \tilde{U}\times\tilde{J} = c\cdot\vec{p}\times\vec{J}$$
$$+ c^2\cdot m\cdot\vec{J} + c^2\cdot n\cdot\vec{p} + i\cdot c\cdot\left(\vec{p}\circ\vec{J} + c^2\cdot n\cdot m\right)$$

The scalar part in the expression for Mechanical Power Density is a Relativistic Bernoulli Pressure and corresponds to the Relativistic Electromagnetic Pressure ($\boldsymbol{A}\circ\boldsymbol{j} + \rho\phi$).

3.1 Thermally extending the Potentials (ϕ) & (A)

The Scalar Potential (ϕ) can be thermally extended. For the moment, let us distinguish two different thermoelectric slopes, the Seebeck slope (α_e) and the Peltier slope (α_i). Now if the thermal conductivity of immobile charge carriers is (κ_e) and the electrical conductivity is (σ_e), the following two equations for current density and energy flux are well known in the literature: Eq 3.1.1a-b

$$\vec{j} = -\sigma_e \cdot \vec{\nabla}(\phi + \alpha_e \cdot T)$$
$$\vec{q} = \vec{j} \cdot (\phi + \alpha_i \cdot T) - \kappa_e \cdot \vec{\nabla} T$$

Observe the adherence of the thermoelectric and electrostatic potentials ($\phi + \alpha T$). To promote the electric field to the dynamic domain, the electromagnetic vector potential (**A**) is needed. Further, to include electromagnetic energy extracted from the charge carriers, we argue, that a Thermo-electro-magnetic Vector-Potential must be added to the Electromagnetic Vector Potential (**A**). The minimal form of such an extension is (**A** + τ_m **grad** αT) where (τ_m) is a material constant and represents the average time a charge carrier spends for momentum exchange with the potentials: Eq 3.1.2a-b

$$\vec{j} = -\sigma_e \cdot \vec{\nabla}(\phi + \alpha_e \cdot T) - \sigma_e \cdot \frac{\partial}{\partial t}\left(\vec{A} + \tau_m \cdot \alpha_e \cdot \vec{\nabla} T\right)$$
$$\vec{q} = \vec{j} \cdot (\phi + \alpha_i \cdot T) - \frac{\kappa_e}{\tau_m \cdot \alpha_e} \cdot \left(\vec{A} + \tau_m \cdot \alpha_e \cdot \vec{\nabla} T\right)$$

The vector (**A**) in the heat flux, eq. 3.1.2b, account for the electromagnetic momentum extracted from the thermal charge carriers. The temporal derivative in the current density, eq. 3.1.2a, account for the momentum exchange in the acceleration and deceleration of charged carriers and is a dynamic extension to Ohms law: "**j** = σ **E** = -σ **grad** ϕ – σ δ**A**/δt". We then thermally extend the potentials "$\phi_T = \phi + \alpha T$" and "$\mathbf{A}_T$ = **A** + $\tau_m \alpha$ **grad** T". To appreciate the connection with Maxwell's Electromagnetic Equations, the thermoelectric multiplier ($\kappa/\tau_m\alpha$) can be expanded by a number of equalities and definitions: Eq 3.1.3

$$\frac{\kappa_e}{\tau_m \cdot \alpha_e} = \frac{\sigma_e \cdot \alpha_e \cdot T_{Ge}}{\tau_m} = \frac{e \cdot \rho_e \cdot \tau_e \cdot \alpha_e \cdot T_{Ge}}{m^* \cdot \tau_m}$$
$$= \rho_e \cdot c_e \cdot T_{Ge} \cdot \left(\frac{\tau_e}{\tau_m}\right) = \rho_e \cdot \bar{v}_m^{\,2}$$

Recognize here a medium or material based temperature ($T_G = \kappa/\alpha^2\sigma = 1/Z$), a mobility related collision-time (τ_e), a charge carrier heat capacity ($c_e = e\alpha_e/m^*$), a charge carrier effective mass (m^*) and an effective charge density (ρ_e). The last equation in fact defines a new thermal velocity material constant ($v_m^2 = c_e\, T_G\, \tau_e/\tau_m$) that is dimensionally equal to a gravitational potential or kinetic energy per mass. The energy flux can now be written as difference of two related terms: Eq 3.1.4

$$\vec{q} = \vec{j} \cdot (\phi + \alpha_i \cdot T) - \rho_e \cdot v_m^{\,2} \cdot \left(\vec{A} + \tau_m \cdot \alpha_e \cdot \vec{\nabla} T\right)$$

This Energy-Flux equation reveals the thermoelectric speed constant (v_m) we use to get a relativistic view on thermoelectromagnetism.

3.2 Relativistic Thermoelectromagnetism

We have now seen that the multiplier ($\kappa/\tau_m\alpha$) to the vector potential (**A**) in the heat flux equation is identical to the Electric Charge Density (ρ_e) times a squared velocity (v_m), unique to the material or medium in question. This enables us to extend the current density vector to a *Photor:* (***j*** = **j** + $i v_m \rho_e$). The fact that v_m is a material constant can be utilized to spatialize the time variable ($v_m t$ = s) and unify the electromagnetic and electrostatic potentials into a *Photor* (v_m**A** + iϕ = **Φ** + iϕ): Eq 3.2.1a-b

$$\vec{j} = -\sigma_e \cdot \vec{\nabla}(\phi + \alpha_e \cdot T) - \sigma_e \cdot \frac{\partial}{\partial t}\left(\vec{A} + \tau_m \cdot \alpha_e \cdot \vec{\nabla} T\right)$$
$$\vec{q} = \vec{j} \cdot (\phi + \alpha_e \cdot T) - \rho_e \cdot v_m^{\,2} \cdot \left(\vec{A} + \tau_m \cdot \alpha_e \cdot \vec{\nabla} T\right)$$

Assuming Seebeck-Peltier symmetry "$\alpha_i = \alpha_e$", both the Current Density (**j**) and the Energy Flux (**q**) can be factored as a Vector-Scalar Cross Product, defined in section 2.2. This is not the only way to do it, but we argue that it is a very efficient notation as shown in the following: Eq 3.2.2a-b

$$\vec{j} + i \cdot \rho_e \cdot v_m$$
$$= -\sigma_e \cdot \left(\vec{\nabla} - \frac{i}{v_m} \cdot \frac{\partial}{\partial t}\right) \times \left(\left(\vec{\Phi} + \alpha_e \cdot \lambda_m \cdot \vec{\nabla} T\right) + i \cdot (\phi + \alpha_e \cdot T)\right)$$

$$\vec{q} + i \cdot p_e \cdot v_m$$
$$= -\left(\vec{j} + i \cdot \rho_e \cdot v_m\right) \times \left(\left(\vec{\Phi} + \alpha_e \cdot \lambda_m \cdot \vec{\nabla} T\right) + i \cdot (\phi + \alpha_e \cdot T)\right)$$

Here we define a collision distance ($\lambda_m = v_m \tau_m$) and an electromagnetic pressure ($p_e = \rho_e \phi_T + \mathbf{A}_T \mathbf{j}$) where the "T" subscript refers to the thermally augmented potentials. The factored equations now read: Eq 3.2.3a-b

$$\tilde{j} = -\sigma_e \cdot \tilde{\nabla} \times \tilde{\Phi}_T$$
$$\tilde{q} = -\tilde{j} \times \tilde{\Phi}_T = -\sigma_e \cdot \tilde{\Phi}_T \times \tilde{\nabla} \times \tilde{\Phi}_T$$

The system (**j**,**q**) is expressed with only one physical material constant (σ_e) and only one thermally augmented potential (Φ_T).

3.3 Thermoelectric Gauge

Now we take a close look at the scalar part of the *Photor* extension to Ohm's law: "**j** + $i v_m \rho_e = \sigma_e$**E** + $i\sigma_e v_m g$" and recall the mobility expression for the electric conductivity ($\sigma_e = \mu_e \rho_e$) to obtain: Eq 3.3.1a-b

$$\rho_e = \sigma_e \cdot \left(\vec{\nabla} \circ \vec{A}_T + \tfrac{1}{v_m^2} \cdot \tfrac{\partial}{\partial t}\phi_T\right)$$
$$= \mu_e \cdot \rho_e \cdot \left(\vec{\nabla} \circ \vec{A}_T + \tfrac{1}{v_m^2} \cdot \tfrac{\partial}{\partial t}\phi_T\right)$$
$$\Leftrightarrow$$
$$\tilde{\nabla} \circ \tilde{A}_T = \vec{\nabla} \circ \vec{A}_T + \tfrac{1}{v_m^2} \cdot \tfrac{\partial}{\partial t}\phi_T = \frac{\rho_e}{\sigma_e} = \frac{1}{\mu_e} = \frac{m^*}{e \cdot \tau_e}$$

This fixes the Thermoelectric Gauge to "g_T = ***div*** $\tilde{A}_T = m^*/e\tau_e$" and shows that the Mass (m) and the Gauge (g) are connected. If the Gauge is assumed zero, that implies a zero mass charged particle, a physical impossibility! All known charges have some mass. A zero Gauge can also be attained if the scattering time tends to infinite. That would constitute a totally free particle.

3.4 A new Dynamic Ohm's Law

We have seen that the Gauge (g) unit is an inverse ***Mobility*** (μ^{-1}). The Gauge is thus ***Confinement***. The SI-unit for (g=div **A**) is [Tesla], just as for the Magnetic Flux Density (**B**=curl **A**). In fact (**B**) and (g) complement each other as the Vector and the Scalar Magnetic Flux Density. Let us summarize the logic: Eq 3.3.2

$$\tilde{j} = \sigma \cdot \tilde{E} \;\Leftrightarrow\; \begin{cases} \vec{j} = \sigma \cdot \vec{E} = \rho \cdot \mu \cdot \vec{E} \\ \rho = \sigma \cdot g = \rho \cdot \mu \cdot g \end{cases} \;\Leftrightarrow\; \begin{cases} \vec{v} = \mu \cdot \vec{E} \\ g = \mu^{-1} \end{cases}$$

We have promoted Ohm's Law into the Vector-Scalar Domain of *Photors* and created a new dynamic Ohm's Law, "$\rho = \sigma g$"

4.1 Thermomagnetic Effects in External Field

Thermal extensions to the Hall effect imply the presence of a magnetic field (B). Define "$\alpha_N = N/R_H$" as the Nernst Thermoelectric Slope, where (N) is the conventional Nernst coefficient and (R_H) is the conventional Hall coefficient. If "$\mu_H = \sigma_H R_H$" is defined as the Hall Mobility, where (σ_H) is the Hall Conductivity, the total thermo-electro-magnetic current is: Eq 4.1.1

$$\vec{j} = \sigma_\Omega \cdot \vec{E} - \sigma_\Omega \cdot \alpha_S \cdot \vec{\nabla}T$$
$$- \mu_H \cdot \vec{B} \times \vec{j} - \mu_H \cdot \alpha_N \cdot \vec{B} \times \vec{\nabla}T$$

The subscripts refer to Ohm, Seebeck, Hall and Nernst respectively. A Lorentz body force ($\boldsymbol{f_H}$) is hidden in this equation and is revealed by re-writing the terms as: Eq 4.1.2

$$\vec{f}_H \equiv \mu_H^{-1} \cdot \left(\vec{j} + \sigma_\Omega \cdot \alpha_S \cdot \vec{\nabla}T\right)$$
$$= \rho_H \cdot \vec{E} + \left(\vec{j} + \sigma_H \cdot \alpha_N \cdot \vec{\nabla}T\right) \times \vec{B}$$

The Hall charge density (ρ_H) is equal to the Coulomb charge density. The relation "$\sigma_H = \sigma_\Omega \tan \theta_H$" defines the Hall Angle (θ_H). It is possible to solve for the Current Density (j), using matrix inversion, and get: Eq 4.1.3

$$\vec{j} = -\sigma_H \cdot \alpha_N \cdot \vec{\nabla}T + \frac{\sigma_\Omega}{1 + \mu_H^2 \cdot B^2} \cdot \vec{E}_T$$
$$- \frac{\mu_H \cdot \sigma_\Omega}{1 + \mu_H^2 \cdot B^2} \cdot \vec{B} \times \vec{E}_T + \frac{\mu_H^2 \cdot \sigma_\Omega}{1 + \mu_H^2 \cdot B^2} \cdot \left(\vec{B} \circ \vec{E}_T\right) \cdot \vec{B}$$

The vector "$\mathbf{E}_T = \mathbf{E} - \alpha_S(1 - \tan \theta_H \tan \theta_N)$**grad** T" is the thermally augmented Electric Field, where "$\tan \theta_N = \alpha_N/\alpha_S$ " is defined as the "Nernst Angle". The denominator ($1+\mu_H^2B^2$) suggests a bell-shaped curve for the "Magneto-conductivity" (ref 5).

4.2 Rationalizing Transport Parameters

Now take a look at the phenomenological heat flux equation, also in external field: Eq 4.2.1

$$\vec{q} = -\kappa \cdot \vec{\nabla}T + \phi \cdot \vec{j} + \alpha_P \cdot T \cdot \vec{j}$$
$$+ \mu_H \cdot \alpha_N \cdot T \cdot \vec{B} \times \vec{j} + \kappa_N \cdot \mu_H \cdot \vec{B} \times \vec{\nabla}T$$

By defining a "Leduc-Righi" thermal conductivity as "$\kappa_N = \alpha_N^2 \sigma_H T$" and using the metallic expression "$\kappa = \alpha_P^2 \sigma_\Omega T$" for the "Fourier-Ficks" thermal conductivity, we can partially factor this expression to: Eq 4.2.2

$$\vec{q} = \phi \cdot \vec{j} + \alpha_P \cdot T \cdot \left(\vec{j} - \sigma_\Omega \cdot \alpha_P \cdot \vec{\nabla}T\right)$$
$$+ \mu_H \cdot \alpha_N \cdot T \cdot \vec{B} \times \left(\vec{j} - \sigma_H \cdot \alpha_N \cdot \vec{\nabla}T\right)$$

The present paper identifies *three* thermoelectric slopes, the Seebeck-slope (α_S), the Peltier-slope (α_P) and the Nernst-slope (α_N).

5.1 Maxwell's Equations Revisited

James Clerk Maxwell related Electric Displacement (**D**), Electric Field (**E**), Electric Polarization (**P**), Magnetic Flux Density (**B**), Magnetic Field Intensity (**H**), Magnetic Induction (**M**), Ampère Current Density ($\mathbf{j_e}$), Electric Charge Density (ρ_e), Magnetic Charge Density (ρ_m) and the four constants (G_0), (G_E), (G_B) and (c): Eq 5.1.1a-d

$$\vec{D} = \tfrac{1}{c} \cdot G_0 \cdot \vec{E} + G_B \cdot \vec{B} + \vec{P}$$
$$\vec{\nabla} \circ \vec{D} = \rho_0 + \vec{\nabla} \circ \vec{P} = \rho_e$$
$$\vec{H} = c \cdot G_0 \cdot \vec{B} + G_E \cdot \vec{E} - \vec{M}$$
$$\vec{\nabla} \circ \vec{H} = \tfrac{G_E}{G_0} \cdot c \cdot \rho_0 - \vec{\nabla} \circ \vec{M} = -c \cdot \rho_m$$

The Speed (c), Conductance (G_0), Resistance "$R_0 = 1/G_0$", Hall-Conductance "$G_H = e^2/2h$", a new Hall-Resistance "$R_H = 1/G_H$" and *The Fine Structure Constant* (α), are closely related: Eq 5.1.2a-b

$$c^{-1} = \sqrt{\varepsilon_0 \cdot \mu_0}, \quad R_0 = \sqrt{\mu_0 / \varepsilon_0} = \alpha \cdot R_H$$

The Greek letters (ε_0) and (μ_0) represent the capacitive like permittivity and the inductive like permeability of vacuum. As an *addition* to the Ampère Current Density ($\mathbf{j_e}$), a trivial *Faraday Current Density* ($\mathbf{j_\gamma = 0}$), and *three* new nonzero

Current Densities ($\mathbf{j_0}$), ($\mathbf{j_p}$) and ($\mathbf{j_m}$) will appear. That enables writing of six Current Density equations: Eq 5.1.3a-f

$$\vec{j}_e = \vec{\nabla} \times \vec{H} - \tfrac{\partial}{\partial t}\vec{D}$$
$$\vec{j}_p = \vec{\nabla} \times \vec{M} + \tfrac{\partial}{\partial t}\vec{P}$$
$$\vec{j}_0 = \tfrac{1}{\mu_0}\cdot\vec{\nabla} \times \vec{B} - \varepsilon_0 \cdot \tfrac{\partial}{\partial t}\vec{E}$$
$$\vec{j}_m = c\cdot\vec{\nabla} \times \vec{D} + \tfrac{1}{c}\cdot \tfrac{\partial}{\partial t}\vec{H}$$
$$\vec{j}_m = c\cdot\vec{\nabla} \times \vec{P} - \tfrac{1}{c}\cdot \tfrac{\partial}{\partial t}\vec{M}$$
$$\vec{0} = G_0\cdot\vec{\nabla} \times \vec{E} + G_0 \cdot \tfrac{\partial}{\partial t}\vec{B}$$

Eq. 5.1.3a is the classical Ampère-Maxwell law and Eq 5.1.3.f is the classical Maxwell-Faraday law, transformed into a current density ($\mathbf{j_\gamma=0}$), and represents the absence of Flux Monopoles. The two ($\mathbf{j_m}$) currents are simply a restatement of the Maxwell-Faraday law. Each divergence of ($\mathbf{j_0}$), ($\mathbf{j_e}$), ($\mathbf{j_p}$) and ($\mathbf{j_m}$) is a continuity equation and the new Charge Density "$\rho_0 = \rho_e + \rho_p$" appears: Eq 5.1.4a-d

$$\vec{\nabla}\circ\vec{j}_e = -\tfrac{\partial}{\partial t}\rho_e \quad ; \quad \vec{\nabla}\circ\vec{j}_0 = -\tfrac{\partial}{\partial t}\rho_0$$
$$\vec{\nabla}\circ\vec{j}_p = -\tfrac{\partial}{\partial t}\rho_p \quad ; \quad \vec{\nabla}\circ\vec{j}_m = -\tfrac{\partial}{\partial t}\rho_m$$

When the vector and scalar Potentials (**A**) and (ϕ), are applied to the Ampère-Maxwell equation, a new current density ($\mathbf{j_0}$), appears, which consists of ($\mathbf{j_e}$) and ($\mathbf{j_p}$). We label this current "The Luminal": Eq 5.1.5

$$\vec{j}_0 = \vec{j}_e + \vec{j}_p = \vec{j}_e + \vec{\nabla}\times\vec{M} + \tfrac{\partial}{\partial t}\vec{P}$$

The Luminal Current Density satisfies an identical equation of continuity as the Ampère Current Density ($\mathbf{j_e}$) does. – Further, if ($\mathbf{j_m}$) is the Monopole Current Density, we identify ($\mathbf{j_p}$) as the Polarization Current Density.

5.2 The Vector and Scalar Potentials (A) and (ϕ)

The absence of Flux Monopoles, or "$\boldsymbol{\nabla \circ B = 0}$", enables us to write the Magnetic Flux Density (**B**) as a curl of a vector potential (**A**) and gives a complete solution for the homogenous equation of (**B**) and (**E**): Eq 5.2.1a-b

$$\vec{E} = -\vec{\nabla}\phi - \tfrac{\partial}{\partial t}\vec{A}, \qquad \vec{B} = \vec{\nabla}\times\vec{A}$$

As the curl of a gradient is zero, the Magnetic Flux Density (**B**), is not altered, if to (**A**), we add a gradient of a scalar. Then (**E**) is invariant if from the Scalar Potential (ϕ), we subtract a corresponding temporal derivative. This constitutes a Gauge Transformation. We construct a Photor from the Vector and Scalar potentials (**A**) & (ϕ): Eq 5.2.2a-d

$$\tilde{A} = \vec{A} + i\cdot\phi/c \qquad \tilde{\nabla}\circ\tilde{A} = g$$
$$\tilde{j} = \vec{j} + i\cdot c\cdot\rho \qquad \tilde{\nabla}\circ\tilde{j} = 0$$

The *Photor* Divergence of ($\tilde{A}$) is the Gauge (g) and the *Photor* Divergence of (***j***) is a Continuity Equation.

5.2 Potentials (A) & (ϕ) Factor Maxwell's Equations

We will now arrive to a pair of Maxwell's equations, with no compromise or ignored terms. The Luminal Current Density ($\mathbf{j}_0$) is Hyperbolic. A new Current Density ($\mathbf{j_0}^*$) is its Elliptic dual. The Charge Density is its own dual, and therefore needs no label: Eq 5.2.1a-d

$$\vec{j}_0 = -\tfrac{1}{\mu_0}\cdot\left(\vec{\nabla}^2\vec{A} - \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}\right) + \tfrac{1}{\mu_0}\cdot\vec{\nabla}\left(\left(\vec{\nabla}\circ\vec{A}\right) + \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$
$$\vec{j}_0^{\,*} = -\tfrac{1}{\mu_0}\cdot\left(\vec{\nabla}^2\vec{A} + \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}\right) + \tfrac{1}{\mu_0}\cdot\vec{\nabla}\left(\left(\vec{\nabla}\circ\vec{A}\right) - \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$
$$\rho_0 = -\varepsilon_0\cdot\left(\vec{\nabla}^2\phi - \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\phi\right) - \varepsilon_0\cdot\tfrac{\partial}{\partial t}\left(\left(\vec{\nabla}\circ\vec{A}\right) + \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$
$$\rho_0^{\,*} = -\varepsilon_0\cdot\left(\vec{\nabla}^2\phi + \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\phi\right) - \varepsilon_0\cdot\tfrac{\partial}{\partial t}\left(\left(\vec{\nabla}\circ\vec{A}\right) - \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$

Here we reveal a symmetric "Wave part" and a "Gauge part". If the "Wave part" is global, the "Gauge part" can be local and seems to ***couple*** the Current to the Charge.

Maxwell's Electromagnetic Equation(s) can be now be written as: Eq 2.3.3

$$\mu_0\cdot\tilde{j}_0 = \tilde{\nabla}g - \tilde{\nabla}^2\tilde{A}$$

6. Conclusion

It has been demonstrated that the Thermoelectric Speed Constant (v_m), defined in eq. 3.1.3, can be used to unite the vector and scalar quantities in thermoelectromagnetism into *Photors*, which greatly simplifies the transport theory as shown in eq. 3.2.3, and provides tools for analytical and explicit solutions to nonlinear and dynamic thermoelectromagnetic problems. In section 4 we inverted the phenomenological thermoelectric current density in eq. 4.1.1, to obtain an explicit equation, eq. 4.1.3, for the current density in an external B-field. Published data on different thermoelectric materials, from different sources, yields to the inequality "$\sigma_\Omega\ \alpha_P >> \sigma_H\ \alpha_N$". In section 5 we expanded and permutated the Maxwell Electromagnetic System and obtained non-compromised general equations, which are most naturally expressed in the Photor Notation introduced in section 2. For clarity, we seldom site explicit the following references as a background:

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