1.1 Electric & Magnetic Entities:

The *seven* Vector Fields of James Clerk Maxwell are: Electric Displacement (**D**), Electric Field (**E**), Electric Polarisation (**P**), Magnetic Flux Density (**B**), Magnetic Field (**H**), Magnetic Induction (**M**) and Ampère Current Density ($\mathbf{j_e}$). Further, *two* Scalar Fields must be defined: Electric Charge Density (ρ_e) and Monopole Charge Density (ρ_m) – giving a total of 14 equations in 23 variables - with *four* Luminal Constants:

$$\vec{D} = \tfrac{1}{c} \cdot G_0 \cdot \vec{E} + G_B \cdot \vec{B} + \vec{P} \qquad \vec{\nabla} \circ \vec{D} = \rho_0 + \vec{\nabla} \circ \vec{P} = \rho_e$$

$$\vec{H} = c \cdot G_0 \cdot \vec{B} + G_E \cdot \vec{E} - \vec{M} \qquad \vec{\nabla} \circ \vec{H} = \tfrac{G_E}{G_0} \cdot c \cdot \rho_0 - \vec{\nabla} \circ \vec{M} = -c \cdot \rho_m$$

Recognize above the Luminal-Speed (c), Luminal-Conductance (G_0) and the static constants (G_E, G_B):

$$c^{-1} = \sqrt{\varepsilon_0 \cdot \mu_0} \qquad G_0 = \sqrt{\varepsilon_0 / \mu_0} = \alpha^{-1} \cdot G_H \qquad R_0 = \sqrt{\mu_0 / \varepsilon_0} = \alpha \cdot R_H$$

Here the Greek letters (ε_0) and (μ_0) represent the capacitive like permittivity and the inductive like permeability of vacuum <u>and</u> "$R_0 = 1/G_0$" is Luminal-Resistance - about 377 Ω, "$G_H = e^2/2h$" is Hall-Conductance, "$R_H = 1/G_H$" is Hall-Resistance and (α) is *The Fine Structure Constant*.

As an *addition* to the Ampère Current Density ($\mathbf{j_e}$), a trivial *Faraday Current Density* ($\mathbf{j_\gamma=0}$), and *four* new Current Densities ($\mathbf{j_0}$), ($\mathbf{j_p}$), ($\mathbf{j_m}$) and ($\mathbf{j_i}$) will be manifested which enables us to write *six* Current Density equations, but until later in *Section 1.14*, we assume that " $G_E = G_B = 0$ " which demands that " $\mathbf{j}_m = \mathbf{j}_i$ ":

$$\vec{j}_e = \vec{\nabla} \times \vec{H} - \tfrac{\partial}{\partial t}\vec{D} \qquad \vec{j}_m = c \cdot \vec{\nabla} \times \vec{D} + \tfrac{1}{c} \cdot \tfrac{\partial}{\partial t}\vec{H}$$

$$\vec{j}_p = \vec{\nabla} \times \vec{M} + \tfrac{\partial}{\partial t}\vec{P} \qquad \vec{j}_i = c \cdot \vec{\nabla} \times \vec{P} - \tfrac{1}{c} \cdot \tfrac{\partial}{\partial t}\vec{M}$$

$$\vec{j}_0 = \tfrac{1}{\mu_0} \cdot \vec{\nabla} \times \vec{B} - \varepsilon_0 \cdot \tfrac{\partial}{\partial t}\vec{E} \qquad \vec{j}_\gamma = G_0 \cdot \vec{\nabla} \times \vec{E} + G_0 \cdot \tfrac{\partial}{\partial t}\vec{B}$$

The top-left equation is the classical Ampère-Maxwell law. The bottom-right is the classical Maxwell-Faraday law, cast into a current density ($\mathbf{j_\gamma=0}$). It represents the absence of a Luminal Monopole Current. The two ($\mathbf{j_m}$) currents are simply a restatement of the Maxwell-Faraday law. Each divergence of ($\mathbf{j_0}$), ($\mathbf{j_e}$), ($\mathbf{j_p}$) and ($\mathbf{j_m}$) is a continuity equation and a new Charge Density "$\rho_0 = \rho_e + \rho_p$" appears here:

$$\vec{\nabla} \circ \vec{j}_e = -\tfrac{\partial}{\partial t}\rho_e \;;\; \vec{\nabla} \circ \vec{j}_0 = -\tfrac{\partial}{\partial t}\rho_0 \;;\; \vec{\nabla} \circ \vec{j}_p = -\tfrac{\partial}{\partial t}\rho_p \;;\; \vec{\nabla} \circ \vec{j}_m = -\tfrac{\partial}{\partial t}\rho_m$$

Observe, that "$\nabla \circ (\mathbf{H}/c) = -\nabla \circ (\mathbf{M}/c)$" represents Monopoles, with ($\mathbf{j_m}$) as the Monopole Current Density.

1.2 The Vector and Scalar Potentials (A) and (ϕ):

The absence of Flux Monopoles, or "$\nabla \circ \mathbf{B} = 0$", enables us to write the Magnetic Flux Density (**B**) as a curl of a vector potential (**A**) and gives a complete solution for the Maxwell-Faraday equation in (**B**) and (**E**):

$$\vec{E} = -\vec{\nabla}\phi - \tfrac{\partial}{\partial t}\vec{A} \qquad \vec{E}^* = -\vec{\nabla}\phi + \tfrac{\partial}{\partial t}\vec{A} \qquad \vec{B} = \vec{\nabla} \times \vec{A}$$

We introduce the field (**E***), which will become useful. As the curl of a gradient is zero, the Magnetic Flux Density (**B**), is not altered, if to (**A**), we add a gradient of a scalar. Then (**E**) is invariant if from the Scalar Potential (ϕ), we subtract a corresponding temporal derivative. This constitutes a Gauge Transformation. For the enthusiastic reader, the conjugate field (E*) reflects the observed *Matter-Antimatter* symmetry.

1.3 The Luminal Current Density (j_0)☺:

When the vector and scalar Potentials (**A**) and (ϕ), are applied to the Ampère-Maxwell equation, a new current density (**j₀**), appear, which consists of (**j_e**) and (**j_p**). We label this current "The Luminal":

$$\vec{j}_0 = \vec{j}_e + \vec{j}_p = \vec{j}_e + \vec{\nabla} \times \vec{M} + \tfrac{\partial}{\partial t}\vec{P} \qquad \vec{\nabla} \circ \vec{j}_0 = -\tfrac{\partial}{\partial t}\rho_0$$

This new current density surprisingly satisfies an identical equation of continuity as the original Ampère's Current Density (**j_e**) did. – Further, if (**j_m**) is the Monopole Current Density, we identify (**j_p**) as the Polarisation Current Density.

We have now arrived to a new pair of Maxwell's equation, with no compromise or ignored terms. The Luminal Current Density (**j_0**) is Hyperbolic. A new Current Density (**j_0^***) is its Elliptic dual. The Charge Density is its own dual, and therefore needs no label:

$$\vec{j}_0 = -\tfrac{1}{\mu_0}\cdot\left(\vec{\nabla}^2\vec{A} - \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}\right) + \tfrac{1}{\mu_0}\cdot\vec{\nabla}\left(\left(\vec{\nabla}\circ\vec{A}\right) + \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$

$$\vec{j}_0^{\,*} = -\tfrac{1}{\mu_0}\cdot\left(\vec{\nabla}^2\vec{A} + \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}\right) + \tfrac{1}{\mu_0}\cdot\vec{\nabla}\left(\left(\vec{\nabla}\circ\vec{A}\right) - \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$

$$\rho_0 = -\varepsilon_0\cdot\left(\vec{\nabla}^2\phi - \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\phi\right) - \varepsilon_0\cdot\tfrac{\partial}{\partial t}\left(\left(\vec{\nabla}\circ\vec{A}\right) + \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right)$$

$$\rho_0^{\,*} = -\varepsilon_0\cdot\left(\vec{\nabla}^2\phi + \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\phi\right) - \varepsilon_0\cdot\tfrac{\partial}{\partial t}\left(\left(\vec{\nabla}\circ\vec{A}\right) - \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi\right) = \rho_0$$

Here we reveal a symmetric "Harmonic Wave part" and a "Gauge Measure part". We will not assign any size preference on either as commonly done in attempts to solve Maxwell's equations. If the "Wave part" is global, the "Gauge part" can be local and should not be fixed - and also seems to ***couple*** the Current to the Charge.

1.4 Maxwell's Equation in the Luminal Domain of Photors:

As a minimal notation for a vector-scalar hybrid, let us extend a vector, just as we extend a real number into the complex domain. Our new extended vector-scalar pair is given the name *Photor* identified with a *tilde*:

$$\tilde{j}_0 = \vec{j}_0 + i\cdot c\cdot\rho_0 \qquad \tilde{A} = \vec{A} + i\cdot\phi/c \qquad \tilde{\nabla} = \vec{\nabla} - i\cdot\tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}$$

Maxwell's EM-equations can now be expressed minimally without presumption on the **P** & **M** fields as:

$$\mu_0\cdot\tilde{j}_0 = \tilde{\nabla}g - \tilde{\nabla}^2\tilde{A} \qquad \tilde{\nabla}\circ\tilde{j}_0 = 0 \qquad \tilde{\nabla}\circ\tilde{A} = g$$

The Gauge (g) is calculated *after* solving the current equation, rather than making any gauge presumption *before* a solution is sought. The gauge will indeed inform us on the nature of a specific problem. By Conjugation, we have three extra variations of the Luminal Maxwell equation as:

$$\mu_0\cdot\tilde{j}_0^{\,*} = \tilde{\nabla}^* g^* - \tilde{\nabla}^2\tilde{A} \qquad \mu_0\cdot\tilde{j}_0 = \tilde{\nabla}g^* - \tilde{\nabla}^2\tilde{A}^* \qquad \mu_0\cdot\tilde{j}_0^{\,*} = \tilde{\nabla}^* g - \tilde{\nabla}^2\tilde{A}^*$$

These equations will be the subject of *Chapter 2* and onwards – but now we need to explore *Photors*.

1.5 Photors and the Luminal domain:

We will now explore the minimal algebra of a vector-scalar hybrid named Photor that we discovered in Maxwell's Electromagnetic Theory. Let us start with the Light-cone-photor "s=**r** + ict". Composition, conjugation and decomposition rules are identical to the rules for the complex plane with "s=x + ict":

$$\tilde{s} = \vec{r} + ict \qquad \vec{r} = \tfrac{1}{2}(\tilde{s} + \tilde{s}^*)$$

$$\tilde{s}^* = \vec{r} - ict \qquad ict = \tfrac{1}{2}(\tilde{s} - \tilde{s}^*)$$

Two scalar products are possible, the internal sum of products and the internal sum of conjugate products, both a real number, the latter positive definite, and a natural choice for the squared Photor-Norm (s^{*2}):

$$\tilde{s} \circ \tilde{s} = \vec{r}^{\,2} - c^2 t^2 \qquad \tilde{s} \circ \tilde{s}^* = \vec{r}^{\,2} + c^2 t^2$$

Observe the Lorentz-Einstein equation for propagation of light signals (s^2) and the positive norm (s^{*2}).
Direct product of two Photors creates a 4x4 object or a matrix, which operate on Photors. We have two:

$$\tilde{s}\tilde{s}^T = \begin{bmatrix} x^2 & xy & xz & ixct \\ yx & y^2 & yz & iyct \\ zx & zy & z^2 & izct \\ ictx & icty & ictz & -c^2t^2 \end{bmatrix} \qquad \tilde{s}^T\tilde{s} = \begin{bmatrix} \tilde{s}^2 & 0 & 0 & 0 \\ 0 & \tilde{s}^2 & 0 & 0 \\ 0 & 0 & \tilde{s}^2 & 0 \\ 0 & 0 & 0 & \tilde{s}^2 \end{bmatrix}$$

The "T" superscript refers to the transpose operation, a diagonal reflection of components.

1.6 The Cross Product and the "Crossed" Product for Photors:

The set of elements in a Photor Product is the linear composition, of all possible pairs, formed in combining all elements of two Photors. To exhaust sign permutations, two will be needed. Name the two operations *Cross Product* (×) and *Crossed Product* (⊗):

$$\tilde{s}\times = \begin{bmatrix} ct & -z & y & ix \\ z & ct & -x & iy \\ -y & x & ct & iz \\ -ix & -iy & -iz & ct \end{bmatrix} \qquad \tilde{s}\otimes = \begin{bmatrix} ct & -z & y & ix \\ z & ct & -x & iy \\ -y & x & ct & iz \\ ix & iy & iz & ct \end{bmatrix}$$

It is easy to verify that " ***s** x **s** =-is²* " is a pure scalar. A Cross Product or a Crossed Product, will be named "Cross-curl" or "Crossed-curl" when the operator is the Photor-Derivative:

$$\tilde{\nabla} \times \tilde{A} = \vec{\nabla} \times \vec{A} + \tfrac{1}{c} \cdot \tfrac{\partial}{\partial t} \vec{A} + \tfrac{1}{c} \cdot \vec{\nabla}\phi + i \cdot \left(\vec{\nabla} \circ \vec{A} + \tfrac{1}{c^2} \cdot \tfrac{\partial}{\partial t} \phi\right)$$

$$\tilde{\nabla} \otimes \tilde{A} = \vec{\nabla} \times \vec{A} + \tfrac{1}{c} \cdot \tfrac{\partial}{\partial t} \vec{A} + \tfrac{1}{c} \cdot \vec{\nabla}\phi - i \cdot \left(\vec{\nabla} \circ \vec{A} - \tfrac{1}{c^2} \cdot \tfrac{\partial}{\partial t} \phi\right)$$

The Magnetic Flux Density, the negative Electric Field and the Gauge appear as a collective entity in both curls. Conjugation and Transposition give us six more curls, total of eight Photor-curls.

1.7 Photor Potentials and Electromagnetic duality:

Momentum and energy considerations show that the vector potential (**A**) is associated with integrated quantities like a charge or a mass, but not to continuous entities like charge density. A "new" *Electric Photor Potential* "**Φ** = c **A**", is measured in Volts, and will be shown to be better candidate. The *Magnetic Photor Potential* "**I** = G_0 **Φ**" is it's dual, measured in Amperes. Invert this and discover a new *Ohm's Law*: "**Φ**= R_0 **I** ". This further justifies defining both Magnetic-and Electric potentials. The Luminal-resistance (R_0) is active in linking the new potentials (**Φ** and **I**). Explore the appearance of the electric field (**E**) and the magnetic flux density (**B**) when expressed from the vectors (**A**, **Φ**, **I**) and scalars (ϕ/c, ϕ, i). The scalar magnetic potential (iota) must not to be confused with the complex number (i̲):

$$\vec{E} = -\vec{\nabla}\phi - \tfrac{\partial}{\partial t}\vec{A} = -\vec{\nabla}\phi - \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{\Phi} = -R_0\cdot\left(\vec{\nabla}\mathbf{i} + \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{I}\right)$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \tfrac{1}{c}\cdot\vec{\nabla}\times\vec{\Phi} = \mu_0\cdot\vec{\nabla}\times\vec{I} \;\Rightarrow\; \vec{H}+\vec{M} = \vec{\nabla}\times\vec{I} + G_E\cdot\vec{E}$$

The new potentials (**Φ**) and (**I**) will be shown to both unify and simplify electromagnetic expressions.

1.8 Extending a 3D-Vector to a Vector-Scalar Hybrid in a 3+1 Domain:

Apart from space-time, currents and potentials, the *Photor* and the *Luminal Domain* can be applied to any physical vector and its most natural scalar. Before we continue it must be acknowledged that the so-called "vector potential" (**A**) is not a potential in the electric sense, but rather a momentum per charge as revealed in the expression "**p**=m**v**+q**A**". This expression can in fact be used as a starting point in the definition of the whole spectrum of physical observable and physical non-observable. The canonical momentum (**p**), when extended to Photor, becomes "**p**+i̲cm". The attachment part consists of the Luminal Speed (c) times the mass (m). This momentum becomes the Einstein energy mc^2 when multiplied with the Luminal Speed:

$$\begin{aligned} &Momentum: && \tilde{p} = \vec{p} + i\cdot c\cdot m && Mom./Charge: && \tilde{A} = \vec{A} + i\cdot\phi/c \\ &Energy: && \tilde{U} = c\cdot\tilde{p} = c\cdot\vec{p} + i\cdot c^2\cdot m && Voltage: && \tilde{\Phi} = c\cdot\tilde{A} = c\cdot\vec{A} + i\cdot\phi \end{aligned}$$

The positive definite norm of the *Energy Photor U* is equal to the relativistic energy for a mass m. A lack of name compels us to use "Momentum per Charge" for the Photor $\tilde{A}$. Voltage on the other hand is recognised as energy per charge. It is now obvious that the vector (c**A**) qualifies to bear the name "Vector Potential". The new *Voltage Potentia*l Photor (**Φ**) multiplied with the *Current Density* Photor (**j**), gives Energy-flux: ☺

$$\begin{aligned} &Current\ Density: && \tilde{j} = \vec{j} + i\cdot c\cdot\rho \\ &Power\ Density: && \tilde{\Phi}\times\tilde{j} = c\cdot\tilde{A}\times\tilde{j} = c\cdot\vec{A}\times\vec{j} + \phi\cdot\vec{j} + c^2\cdot\rho\cdot\vec{A} + i\cdot c\cdot\left(\vec{A}\circ\vec{j} + \rho\cdot\phi\right) \end{aligned}$$

The attachment part in the expression for Electromagnetic Power Density must be identified with *pressure*. That can be seen when compared with a particle-flux and energy in mechanics. To do so, we need identical particles of mass (m) and particle density (n) of identical particles:

$$\begin{aligned} &Matter-flux: && \tilde{J} = \vec{J} + i\cdot c\cdot n \\ &Power\ Density: && \tilde{U}\times\tilde{J} = c\cdot\vec{p}\times\vec{J} + c^2\cdot m\cdot\vec{J} + c^2\cdot n\cdot\vec{p} + i\cdot c\cdot\left(\vec{p}\circ\vec{J} + c^2\cdot n\cdot m\right) \end{aligned}$$

The attachment part in the expression for Mechanical Power Density is a Relativistic Bernoulli Pressure and corresponds to the Relativistic Electromagnetic Pressure (***A*** ∘ ***j***).

1.9 The Luminal Current Density and Flux of Probabilities:

The *Electric Photor Potential* "$\mathbf{\Phi} = c\,\mathbf{A} = R_0\,\mathbf{I}$" and it's dual, the *Magnetic Photor Potential* "$\mathbf{I} = G_0\,\mathbf{\Phi} = G_0\,c\,\mathbf{A}$", defined in section 1.7, can simplify many complex problems. To see how, start by writing the vacuum permeability (μ_0) in terms of the *Fine Structure Constant* (α) and the most fundamental physical constants (α, e, c & h):

$$\tilde{j}_0 = \mu_0^{-1} \cdot \left(\tilde{\nabla} g - \tilde{\nabla}^2 \tilde{A}\right) = \frac{e^2 \cdot c}{2\alpha \cdot h} \cdot \left(\tilde{\nabla}\left(\tilde{\nabla} \circ \tilde{A}\right) - \tilde{\nabla}^2 \tilde{A}\right)$$

The Luminal Speed (c) and the Luminal Conductance "$G_0 = e^2/2\alpha h$" factor this into "$G_0\, c\mathbf{A} = G_0\,\mathbf{\Phi} = \mathbf{I}$". Instead of defining a new function for (cg), we use the full expression "$cg = \nabla \circ \mathbf{\Phi}$":

$$\tilde{j}_0 = G_0 \cdot \left(\tilde{\nabla}\left(\tilde{\nabla} \circ \tilde{\Phi}\right) - \tilde{\nabla}^2 \tilde{\Phi}\right) = \left(\tilde{\nabla}\tilde{\nabla} \circ \tilde{I} - \tilde{\nabla} \circ \tilde{\nabla}\tilde{I}\right)$$

As the dot product has higher precedence, parenthesis can be dropped in the last equality. The current density (j_0) is a second order spatial derivative of a Photor Potential ($\hat{\boldsymbol{I}}$). The last equation reminds one of the probability current density (***J***) in quantum mechanics. Both currents have a non-commuting product. To see the similarity, let (ψ) be a normalised wave function measured in [$m^{-2/3}$]. The probability current in quantum mechanics, measured in [Hz/m^2], reads:

$$\vec{J}_{\psi,m} = \frac{h}{2\pi \cdot i \cdot m} \cdot \left(\psi^* \vec{\nabla} \psi - \psi \vec{\nabla} \psi^*\right) = \frac{1}{m} \cdot \left(\psi^* \vec{p} \psi - \psi \vec{p} \psi^*\right)$$

We transformed the momentum operator back into a vector, and the wave function into an operator. The mass (m) is an integrated quantity, just as the charge (q), and will give singularity-problems. One possible solution is to convert the momentum "$\mathbf{p}=m\mathbf{v}+q\mathbf{A}$" into a momentum density which is equal to a matter flux [$Kg/s\, m^2$]. To underline the symmetry and duality of the electromagnetic term ($q\mathbf{A}$) and the mechanical term ($m\mathbf{v}$), look at the probability fluxes in [Hz/m^2] reduced to a *Frequency Potentials* "$\boldsymbol{f}_A = \mathbf{I}/q$" and "$\boldsymbol{f}_v = \mathbf{v}/\lambda$":

$$\vec{J}_{v,m} = \left(\psi^* \vec{v} \psi - \psi \vec{v} \psi^*\right) = \left(\tilde{\nabla}\tilde{\nabla} \circ \tilde{f}_v - \tilde{\nabla} \circ \tilde{\nabla}\tilde{f}_v\right)$$
$$\vec{J}_{A,q} = \left(\chi^* \vec{A} \chi - \chi \vec{A} \chi^*\right) = \left(\tilde{\nabla}\tilde{\nabla} \circ \tilde{f}_A - \tilde{\nabla} \circ \tilde{\nabla}\tilde{f}_A\right)$$

For consistent units, the electromagnetic current density operator is normalised by ($\chi^*\chi$)~(q/m), but *not unity* as the particle probability density ($\psi^*\psi$)~(1). The thought cannot escape, that Quantum Physics needs the lost *Gauge* in Maxwell's Physics. The difficulty of using (q, **A**) and (m, **v**) in field equations is illustrated and verified here. Neutral flux ($J_{v,m}$) and charged flux ($J_{A,q}$) is both derivable from a *Frequency Potential* (***f***). The two new frequency potentials "$\boldsymbol{f}_A = \mathbf{I}/q$" and "$\boldsymbol{f}_v = \mathbf{v}/\lambda$" relate the charge (q) to *a wavelength* (λ)!

The connection to Quantum Mechanics is now present. We subsequently define the *Image Derivative*:

$$\left(\tilde{\nabla}/i\right)\tilde{A} = \left(\tfrac{-1}{c}\tfrac{\partial}{\partial t} - i \cdot \vec{\nabla}\right)\left(\vec{A} + i \cdot \phi/c\right) = \tfrac{-1}{c} \cdot \tfrac{\partial}{\partial t}\vec{A} + \tfrac{1}{c} \cdot \vec{\nabla}\phi - i \cdot g = \tfrac{-1}{c} \cdot \vec{E}^* - i \cdot g$$
$$\left(\tilde{\nabla}^*/i\right)\tilde{A} = \left(\tfrac{1}{c}\tfrac{\partial}{\partial t} - i \cdot \vec{\nabla}\right)\left(\vec{A} + i \cdot \phi/c\right) = \tfrac{1}{c} \cdot \tfrac{\partial}{\partial t}\vec{A} + \tfrac{1}{c} \cdot \vec{\nabla}\phi - i \cdot g^* = \tfrac{-1}{c} \cdot \vec{E} - i \cdot g^*$$

This is to be compared to the momentum operator in quantum mechanics "$\boldsymbol{p} = -i\hbar\nabla$" which is also a complex operator operating on a complex operand.

1.10 Coulomb's Potential from Plank's Quantum of Action:

The *Coulomb Scalar Potential* (ϕ_C) was discovered a century before Planck's Action - and the subsequent development of Quantum Physics. When the Coulomb potential is extended to dynamical and relativistic effects, it is known as the Lienard-Wiechert Scalar Potential. To dig into, we need a scalar frequency "$f_c=c/2\pi r$" and a vector frequency "$\mathbf{f}_v=\mathbf{v}/2\pi r$", both well defined for ($|\mathbf{r}|>0$):

$$\phi_C = \frac{e\cdot c}{4\pi\cdot\varepsilon_0\cdot(c\cdot r-\vec{v}\circ\vec{r})} = \frac{\alpha\cdot h\cdot c}{2\pi\cdot e\cdot(r-\vec{\beta}\circ\vec{r})} = \left(\frac{\alpha\cdot h\cdot f_c}{e}\right)\cdot\frac{1}{1-\vec{\beta}\circ\vec{\kappa}_r}$$

$$\vec{A}_C = \frac{e\cdot\vec{v}}{4\pi\cdot\varepsilon_0\cdot c\cdot(c\cdot r-\vec{v}\circ\vec{r})} = \frac{\alpha\cdot h\cdot\vec{v}}{2\pi\cdot e\cdot c\cdot(r-\vec{\beta}\circ\vec{r})} = \frac{1}{c}\cdot\left(\frac{\alpha\cdot h\cdot f_c}{e}\right)\cdot\frac{\vec{\beta}}{1-\vec{\beta}\circ\vec{\kappa}_r}$$

The Fine Structure Constant (α) is seen to connect the Coulomb Potential Energy "$E=e\phi$" to Planck's Photon Energy "$E=hf$". Kappa (κ) is here a normalised position vector "$\boldsymbol{\kappa}=\mathbf{r}/r$" and beta is the velocity ratio "$\beta = \mathbf{v}/c$". By extending the velocity-ratio to a Photor "$\boldsymbol{\beta} = \mathbf{v}/c + i$" it is apparent, that the scalar and vector potentials can be combined into a *Vector-Scalar-Hybrid*; just multiply (**A**) with (c) and add ($i\phi$). This will indeed give the *Coulomb Photor Potential* "$\boldsymbol{\Phi}=c\mathbf{A}+i\phi$":

$$\tilde{\Phi}_C = c\cdot\vec{A}_C + i\cdot\phi_C = \left(\frac{\alpha\cdot h\cdot f_c}{e}\right)\cdot\frac{\vec{\beta}+i}{1-\vec{\beta}\circ\vec{\kappa}_r} = \left(\frac{\alpha\cdot h\cdot f_c}{e}\right)\cdot\frac{\tilde{\beta}}{1-\vec{\beta}\circ\vec{\kappa}_r} = \left(\frac{\alpha}{e}\right)\cdot h\cdot\tilde{f}'$$

We have reduced the Coulomb Photor Potential ($\boldsymbol{\Phi}$) to an expression involving only the Frequency Photor ($\boldsymbol{f}'$) and three fundamental constants.

$$\tilde{f}' = f_r\cdot\frac{\vec{\beta}+i}{1-\vec{\beta}\circ\vec{\kappa}} = \left(\frac{1}{2\pi\cdot r}\right)\cdot\frac{\vec{v}+i\cdot c}{1-\vec{\beta}\circ\vec{\kappa}} = \frac{\tilde{v}}{2\pi\cdot r'} = \frac{\tilde{v}}{\lambda'}$$

1.11 Coulomb's Potential from Einstein's Energy & Momentum:

The Classical Radius of the Electron charge (+e or -e) is "$r_e = \alpha h/2\pi m_e c = \alpha\hbar/m_e c$". The heavier the charge, the shorter it's radius! Acknowledge that this radius is independent from the concept of *Charge*, but applies equally to charged and uncharged states of matter. The Scalar and Vector Coulomb Potentials are derived from Energy (mc^2) and Momentum (**p**) as shown here:

$$\phi_C = \left(\frac{\alpha\cdot h\cdot c}{2\pi\cdot r_e\cdot e}\right)\cdot\left(\frac{r_e}{r}\right)\cdot\frac{1}{1-\vec{\beta}\circ\vec{\kappa}_r} = \left(\frac{m_e\cdot c^2}{e}\right)\cdot\frac{r_e}{r-\vec{\beta}\circ\vec{r}} = \left(\frac{r_e}{e}\right)\cdot\frac{c^2\cdot m_e}{r'}$$

$$\vec{\Phi}_C = \left(\frac{\alpha\cdot h\cdot c}{2\pi\cdot r_e\cdot e}\right)\cdot\left(\frac{r_e}{r}\right)\cdot\frac{\vec{\beta}}{1-\vec{\beta}\circ\vec{\kappa}_r} = \left(\frac{m_e\cdot c\cdot\vec{v}}{e}\right)\cdot\frac{r_e}{r-\vec{\beta}\circ\vec{r}} = \left(\frac{r_e}{e}\right)\cdot\frac{c\cdot\vec{p}_e}{r'}$$

The potentials change sign if ($\beta>1$). We conclude that the classical energy radius (r_e) is the radius where interaction strength goes beyond the creation energy "$E=m_ec^2$" for the elementary charge (e) in question.

1.12 The Potential (A) reduced to Reciprocal Length (k):

The "*Fine Structure Constant*", is physically equivalent to our Luminal Resistance (R_0) by way of the fundamental constants (α), (e) and (h), as seen in the expression "$R_0 = 2h\alpha/e^2$". We benefit from defining a new Photor (***k***), having the physical dimension of reciprocal length. We do so, by multiplying our faithful Photor Potential (***A***) with the ratio "$2\pi e/h$". This defines a new vector (***k***) and a scalar (ω):

$$\tilde{k} = \vec{k} + i\cdot\omega/c \qquad \vec{k} = e\cdot\vec{A}/\alpha\cdot\hbar \qquad \omega = e\cdot\phi/\alpha\cdot\hbar$$

To illustrate with an example: if "$\phi = e/4\pi\varepsilon r$" is the Coulomb Potential, then the transformation would be: "$\boldsymbol{k} = \boldsymbol{v}/cr$" and "$\omega = c/r$". An atomic view into (***E***) and (***B***) is obtained when expressed from (***k***) & (ω):

$$\vec{E} = -\tfrac{\alpha\cdot\hbar}{e}\cdot\left(\vec{\nabla}\omega + \tfrac{\partial}{\partial t}\vec{k}\right) \qquad \vec{B} = \tfrac{\alpha\cdot\hbar}{e}\cdot\vec{\nabla}\times\vec{k} \qquad \tilde{\nabla}\circ\tilde{k} = e\cdot g/\alpha\cdot\hbar = n$$

The Photor divergence of (***k***) has a physical dimension of reciprocal area. We name this divergence (n) and it can replace our old friend, *The Gauge* (g). The Charge Density (ρ_0), the Luminal Current Density (j_0) and the Photor Force "$f_0 = j_0 \times \nabla \times A$" (defined in section 3.1), take on an impressive form when expressed from (***k***) and (ω):

$$\rho_0 = \tfrac{-e}{4\pi\cdot c}\cdot\left(\tilde{\nabla}^2\omega + \tfrac{\partial}{\partial t}n\right) \qquad \tilde{j}_0 = \tfrac{e\cdot c}{4\pi}\cdot\left(\tilde{\nabla}n - \tilde{\nabla}^2\tilde{k}\right) \qquad \tilde{f}_0 = \tfrac{\alpha\cdot h\cdot c}{8\pi^2}\cdot\left(\tilde{\nabla}n - \tilde{\nabla}^2\tilde{k}\right)\times\tilde{\nabla}\times\tilde{k}$$

The observant reader will have noticed that the electric and magnetic fields have been normalised with reference to *The Quantum of Extra-nuclear Flux* "$(\boldsymbol{B}\circ d\boldsymbol{S})_{Bohr} = \alpha h/2\pi e$". This is in accordance with the fact, that the domain of action for electric and magnetic fields are extra-nuclear or atomic or inter-atomic.

1.13 Energy Density for the Electromagnetic Fields and Fluxes:

The task is to empty the permutations of the energy in terms of different pares of electromagnetic entities (**D**, **E**, **P**, **B**, **H** & **M**). That will give us the minimal expression for the total and or partial energy densities. Let us start with following nine definitions:

$$u_{D,B} = \frac{1}{2\cdot\varepsilon_0}\cdot\left(D^2 + G_0{}^2\cdot B^2\right) \; ; \quad u_{P,B} = \frac{1}{2\cdot\varepsilon_0}\cdot\left(P^2 + G_0{}^2\cdot B^2\right) \; ; \quad u_{E,B} = \frac{\varepsilon_0}{2}\cdot\left(E^2 + c^2\cdot B^2\right)$$

$$u_{D,H} = \frac{1}{2\cdot\varepsilon_0}\cdot\left(D^2 + \frac{1}{c^2}\cdot H^2\right) \; ; \quad u_{P,H} = \frac{1}{2\cdot\varepsilon_0}\cdot\left(P^2 + \frac{1}{c^2}\cdot H^2\right) \; ; \quad u_{E,H} = \frac{\varepsilon_0}{2}\cdot\left(E^2 + R_0{}^2\cdot H^2\right)$$

$$u_{D,M} = \frac{1}{2\cdot\varepsilon_0}\cdot\left(D^2 + \frac{1}{c^2}\cdot M^2\right) \; ; \quad u_{P,M} = \frac{1}{2\cdot\varepsilon_0}\cdot\left(P^2 + \frac{1}{c^2}\cdot M^2\right) \; ; \quad u_{E,M} = \frac{\varepsilon_0}{2}\cdot\left(E^2 + R_0{}^2\cdot M^2\right)$$

The Luminal-Speed (c), Luminal-Conductance (G_0), Luminal-Resistance "$R_0 = 1/G_0$" were earlier defined in section 1.1, still we repeat them for convenience:

$$c^{-1} = \sqrt{\varepsilon_0\cdot\mu_0} \qquad G_0 = \sqrt{\varepsilon_0/\mu_0} = G_H/\alpha \qquad R_0 = \sqrt{\mu_0/\varepsilon_0} = \alpha\cdot R_H$$

Here as before, "$G_H = e^2/2h$" is the Hall-Conductance, "$R_H = 1/G_H$" is the Hall-Resistance and (α) is *The Fine Structure Constant.* Having the choice of 8 Constants ($c, \varepsilon_0, \mu_0, \alpha, G_0, R_0, G_H, R_H$) enables us to express any electromagnetic statement, with the smallest amount of constants.

1.14 Energy and Momentum Transport with Electromagnetic Entities:

To clarify in what each energy component in electromagnetism stands for, let us write the two most popular vector products of the electric displacement (**D**=ε_0**E**+**P**) measured in [C/m^2], and magnetic (**B**=μ_0**H**+μ_0**M**) entities:

$$\vec{\nabla}\circ\left(\vec{B}\times\vec{E}\right) = E\circ(\nabla\times B)-B\circ(\nabla\times E) = \mu_0\cdot\vec{E}\circ\left(\vec{j}_e+\frac{\partial}{\partial t}\vec{P}\right)+\frac{1}{2}\cdot\frac{\partial}{\partial t}\left(\frac{1}{c^2}\cdot E^2+B^2\right)+\vec{E}\circ\vec{\nabla}\times\vec{M}$$

$$\vec{\nabla}\circ\left(\vec{H}\times\vec{E}\right)= E\circ(\nabla\times H)-H\circ(\nabla\times E)=\vec{E}\circ\left(\vec{j}_e+\frac{\partial}{\partial t}\vec{P}\right)+\frac{1}{2\cdot\mu_0}\cdot\frac{\partial}{\partial t}\left(\frac{1}{c^2}\cdot E^2+B^2\right)-\vec{M}\circ\frac{\partial}{\partial t}\vec{B}$$

Both expressions show energy density from the electromagnetic fields (**E**, **B**). To get a still-better look, compare with the six currents from the extended set of Maxwell's equation from *Section 1.1*:

$$\vec{j}_e=\vec{\nabla}\times\vec{H}-\frac{\partial}{\partial t}\vec{D} \qquad \vec{\nabla}\circ\vec{j}_e=-\frac{\partial}{\partial t}\rho_e \qquad \vec{\nabla}\circ\vec{D}=\rho_e$$

$$\vec{j}_p=\vec{\nabla}\times\vec{M}+\frac{\partial}{\partial t}\vec{P} \qquad \vec{\nabla}\circ\vec{j}_p=-\frac{\partial}{\partial t}(\rho_0-\rho_e) \qquad \vec{\nabla}\circ\vec{P}=-(\rho_0-\rho_e)$$

$$\vec{j}_0=\tfrac{1}{\mu}\cdot\vec{\nabla}\times B-\varepsilon\frac{\partial}{\partial t}E \qquad \vec{\nabla}\circ\vec{j}_0=-\frac{\partial}{\partial t}\rho_0 \qquad \vec{\nabla}\circ E=\frac{1}{\varepsilon}\cdot\rho_0$$

$$\vec{j}_m=c\cdot\vec{\nabla}\times\vec{D}+\tfrac{1}{c}\cdot\frac{\partial}{\partial t}\vec{H} \qquad \vec{\nabla}\circ\vec{j}_m=-\frac{\partial}{\partial t}\rho_m \qquad \vec{\nabla}\circ\vec{H}=-c\cdot\rho_m$$

$$\vec{j}_i=c\cdot\vec{\nabla}\times\vec{P}-\tfrac{1}{c}\cdot\frac{\partial}{\partial t}\vec{M} \qquad \vec{\nabla}\circ\vec{j}_i=-\frac{\partial}{\partial t}\rho_i \qquad \vec{\nabla}\circ\vec{M}=c\cdot\rho_i$$

$$\vec{j}_\gamma=\sqrt{\tfrac{\varepsilon}{\mu}}\cdot\vec{\nabla}\times\vec{E}+\sqrt{\tfrac{\varepsilon}{\mu}}\cdot\frac{\partial}{\partial t}\vec{B} \qquad \vec{\nabla}\circ\vec{j}_\gamma=0 \qquad \vec{\nabla}\circ\vec{B}=\sqrt{\tfrac{\mu}{\varepsilon}}\cdot 0=0$$

Only the 1st Current Density is widely known - the Ampere Current Density (**j**$_e$) – while the last current (**j**$_\gamma$) is implicitly zero by the Faraday-Maxwell equation. Beside the redundant "Luminal Monopole Current" (**j**$_\gamma$) you see four new and unique Current Densities. Now evaluate the cross product of all possible pares of electric and magnetic entities:

$$\frac{1}{\varepsilon}\cdot\vec{\nabla}\circ\left(\vec{H}\times\vec{D}\right) =\frac{1}{\varepsilon}\cdot\vec{D}\circ\vec{j}_e-\frac{1}{c\cdot\varepsilon}\cdot\vec{H}\circ\vec{j}_m+\frac{1}{2\cdot\varepsilon}\cdot\frac{\partial}{\partial t}\left(D^2+\frac{1}{c^2}\cdot H^2\right)=\frac{1}{\varepsilon}\cdot\vec{D}\circ\vec{j}_e-\frac{1}{c\cdot\varepsilon}\cdot\vec{H}\circ\vec{j}_m+\frac{\partial}{\partial t}u_{D,H}$$

$$\frac{1}{\varepsilon}\cdot\vec{\nabla}\circ\left(\vec{M}\times\vec{P}\right) =\frac{1}{\varepsilon}\cdot\vec{P}\circ\vec{j}_p-\frac{1}{c\cdot\varepsilon}\cdot\vec{M}\circ\vec{j}_m+\frac{1}{2\cdot\varepsilon}\cdot\frac{\partial}{\partial t}\left(P^2+\frac{1}{c^2}\cdot M^2\right)=\frac{1}{\varepsilon}\cdot\vec{P}\circ\vec{j}_p-\frac{1}{c\cdot\varepsilon}\cdot\vec{M}\circ\vec{j}_m+\frac{\partial}{\partial t}u_{P,M}$$

$$\frac{1}{\mu}\cdot\vec{\nabla}\circ\left(\vec{B}\times\vec{E}\right) =\vec{E}\circ\vec{j}_0+\frac{1}{2\cdot\mu}\cdot\frac{\partial}{\partial t}\left(\frac{1}{c^2}\cdot E^2+B^2\right)=\vec{E}\circ\vec{j}_0+\frac{\partial}{\partial t}u_{E,B}$$

From *Sec 1.13*, we can see that the three energy densities where "$u_{D,H} = u_{P,M} + u_{E,B}$ + (**E**∘**P** - **B**∘**M**)": To refresh readers memory, the *Luminal Current Density* (**j**$_0$) appears when Maxwell's Equation are expressed in terms of the fields (**E**) and (**B**) only:

$$\vec{\nabla}^2\vec{E}-\frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}\vec{E}=\mu_0\cdot\left(\frac{\partial}{\partial t}\vec{j}_0+c^2\cdot\vec{\nabla}\rho_0\right)=R_0\cdot\left(\tilde{\nabla}^*/i\right)\cdot\tilde{j}_0$$

$$\vec{\nabla}^2\vec{B}-\frac{1}{c^2}\cdot\frac{\partial^2}{\partial t^2}\vec{B}=-\mu_0\cdot\vec{\nabla}\times\left(\vec{j}_e+\frac{\partial}{\partial t}\vec{P}+\vec{\nabla}\times\vec{M}\right)=-(R_0/c)\cdot\vec{\nabla}\times\vec{j}_0$$

The Luminal Current Density is present here, and a new usage is for the Image Derivative from *Sec 1.9*.

1.15 Ohm's law extended to Photors at finite Temperature (T):

When the current density vector (j) is extended to a *Photor:* "$\boldsymbol{j} = \mathbf{j} + i v_m \rho_e$" the velocity v_m can be a material constant which can further be utilized to spatialize the time variable "$v_m t = s$" and to combine the electromagnetic and electrostatic potentials into a vector-scalar hybrid, which is in our case represented by the *Photor* "$v_m\mathbf{A} + i\phi = \mathbf{\Phi} + i\phi$". Now extended the potentials to include thermal effects "$\phi_T = \phi + \alpha_e T$":

$$\vec{j} = -\sigma_e \cdot \vec{\nabla}(\phi + \alpha_e \cdot T) - \sigma_e \cdot \frac{\partial}{\partial t}\left(\vec{A} + \tau_m \cdot \alpha_e \cdot \vec{\nabla} T\right)$$
$$\vec{q} = \vec{j} \cdot (\phi + \alpha_e \cdot T) - \rho_e \cdot v_m^{\,2} \cdot \left(\vec{A} + \tau_m \cdot \alpha_e \cdot \vec{\nabla} T\right)$$

$$\Leftrightarrow$$

$$\vec{j} + i \cdot \rho_e \cdot v_m = -\sigma_e \cdot \left(\vec{\nabla} - \frac{i}{v_m} \cdot \frac{\partial}{\partial t}\right) \times \left(\left(\vec{\Phi} + \alpha_e \cdot \lambda_m \cdot \vec{\nabla} T\right) + i \cdot (\phi + \alpha_e \cdot T)\right)$$
$$\vec{q} + i \cdot p_e \cdot v_m = -\left(\vec{j} + i \cdot \rho_e \cdot v_m\right) \times \left(\left(\vec{\Phi} + \alpha_e \cdot \lambda_m \cdot \vec{\nabla} T\right) + i \cdot (\phi + \alpha_e \cdot T)\right)$$

Here (α_e) is the thermo-potential slope and (τ_m) is the time between collisions of the travelling charges. To simplify expressions, we have defined a new collision distance "$\lambda_m = v_m \tau_m$" and a new "scalar" (p_e), the electromagnetic pressure "$p_e = \rho_e \phi_T + \mathbf{A}_T \circ \mathbf{j}$" where the "T" subscript refers to the thermally augmented potentials.

$$\tilde{j} = -\sigma_e \cdot \tilde{\nabla} \times \tilde{\Phi}_T$$
$$\tilde{q} = -\tilde{j} \times \tilde{\Phi}_T = -\sigma_e \cdot \tilde{\Phi}_T \times \tilde{\nabla} \times \tilde{\Phi}_T$$

This statement may seem too simple to be true! But we have in fact managed to reduce a dynamical and dissipative system (**j**,**q**), into two expressions involving only <u>one</u> physical constant (σ_e) and only <u>one</u> thermally augmented 4-potential (Φ_T)!

The scalar part of our newly obtained extension to Ohm's-law is "$v\rho = \sigma_e \, div \, A_T$" and recalling the mobility expression for electric charge conductivity ($\sigma_e = \mu_e \rho_e$) - we easily obtain:

$$\rho_e = \sigma_e \cdot \left(\vec{\nabla} \circ \vec{A}_T + \tfrac{1}{v_m^{\,2}} \cdot \tfrac{\partial}{\partial t} \phi_T\right) = \mu_e \cdot \rho_e \cdot \left(\vec{\nabla} \circ \vec{A}_T + \tfrac{1}{v_m^{\,2}} \cdot \tfrac{\partial}{\partial t} \phi_T\right)$$
$$\Leftrightarrow$$
$$\vec{\nabla} \circ \vec{A}_T + \tfrac{1}{v_m^{\,2}} \cdot \tfrac{\partial}{\partial t} \phi_T = \frac{\rho_e}{\sigma_e} = \frac{1}{\mu_e} = \frac{m}{e \cdot \tau_e} = \tilde{\nabla} \circ \tilde{A}_T = g_T$$

We have fixed the Thermoelectric Gauge to "$g = div \, A = m/e\tau$" - and displayed a connection between the Mass (m) and the Gauge (g). In other words, if the Gauge is assumed zero, that implies a zero mass charged particle, a physical impossibility! All known charges have some mass. A zero Gauge can also be attained if the scattering time tends to infinite. That would constitute a totally free particle. We have seen before, that the Gauge (g) unit is an inverse ***<u>Mobility</u>*** (μ^{-1}). The Gauge is thus ***<u>Confinement</u>***. The SI-unit for "g=div **A**" is [Tesla], just as for the Magnetic Flux Density "**B**=curl **A**". In fact (**B**) and (g) complement each other as the Vector and the Scalar Magnetic Flux Density. Let us summarize the logic:

$$\tilde{j} = \sigma \cdot \tilde{E} \quad \Leftrightarrow \quad \begin{cases} \vec{j} = \sigma \cdot \vec{E} = \rho \cdot \mu \cdot \vec{E} \\ \rho = \sigma \cdot g = \rho \cdot \mu \cdot g \end{cases} \quad \Leftrightarrow \quad \begin{cases} \vec{v} = \mu \cdot \vec{E} \\ g = \mu^{-1} \end{cases}$$

We have promoted Ohm's Law into the Vector-Scalar Domain of Photors - and discovered a new dynamic Ohm's Law: "$\rho = \sigma g$", thus connecting charge density to conductivity.