2.1 Cross-Curls and Crossed-Curls:

Consider all possibilities of combining ***Circulation*** = left or right *curl*, ***Radiance*** = in or out *divergence* and ***Polarity*** = positive or negative *charge*. For an example, you could have "Right-Out-Negative" or "Left-In-Positive". Forming all combination give us 2^3=8 possibilities. Conjugating the operand doubles that number and adds the fourth dynamic attribute giving a total of 2^4=16 curls:

Cross-curl of the Photor Potential and its Conjugate:

$$\tilde{\nabla} \times \tilde{A} = \vec{B} - \tfrac{1}{c}\cdot\vec{E} + i\cdot g \qquad \tilde{\nabla}^* \times \tilde{A}^* = \vec{B} - \tfrac{1}{c}\cdot\vec{E} - i\cdot g$$

$$\tilde{\nabla}^* \times \tilde{A} = \vec{B} + \tfrac{1}{c}\cdot\vec{E}^* - i\cdot g^* \qquad \tilde{\nabla} \times \tilde{A}^* = \vec{B} + \tfrac{1}{c}\cdot\vec{E}^* + i\cdot g^*$$

$$\tilde{\nabla}^T \times \tilde{A} = -\vec{B} + \tfrac{1}{c}\cdot\vec{E}^* - i\cdot g^* \qquad \tilde{\nabla}^A \times \tilde{A}^* = -\vec{B} + \tfrac{1}{c}\cdot\vec{E}^* + i\cdot g^*$$

$$\tilde{\nabla}^A \times \tilde{A} = -\vec{B} - \tfrac{1}{c}\cdot\vec{E} + i\cdot g \qquad \tilde{\nabla}^T \times \tilde{A}^* = -\vec{B} - \tfrac{1}{c}\cdot\vec{E} - i\cdot g$$

Crossed-curl of the Photor Potential and it's Conjugate:

$$\tilde{\nabla} \otimes \tilde{A} = \vec{B} - \tfrac{1}{c}\cdot\vec{E} - i\cdot g^* \qquad \tilde{\nabla}^* \otimes \tilde{A}^* = \vec{B} - \tfrac{1}{c}\cdot\vec{E} + i\cdot g^*$$

$$\tilde{\nabla}^* \otimes \tilde{A} = \vec{B} + \tfrac{1}{c}\cdot\vec{E}^* + i\cdot g \qquad \tilde{\nabla} \otimes \tilde{A}^* = \vec{B} + \tfrac{1}{c}\cdot\vec{E}^* - i\cdot g$$

$$\tilde{\nabla}^T \otimes \tilde{A} = -\vec{B} - \tfrac{1}{c}\cdot\vec{E} - i\cdot g^* \qquad \tilde{\nabla}^A \otimes \tilde{A}^* = -\vec{B} - \tfrac{1}{c}\cdot\vec{E} + i\cdot g^*$$

$$\tilde{\nabla}^A \otimes \tilde{A} = -\vec{B} + \tfrac{1}{c}\cdot\vec{E}^* + i\cdot g \qquad \tilde{\nabla}^T \otimes \tilde{A}^* = -\vec{B} + \tfrac{1}{c}\cdot\vec{E}^* - i\cdot g$$

Cross-curl of a Photor Gradient and its Conjugate:

$$\tilde{\nabla} \times \tilde{\nabla}\phi = i\cdot\tilde{\nabla}^2\phi \qquad \tilde{\nabla}^* \times \tilde{\nabla}^*\phi = -i\cdot\tilde{\nabla}^2\phi$$

$$\tilde{\nabla}^* \times \tilde{\nabla}\phi = \tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi - i\cdot\tilde{\nabla}\circ\tilde{\nabla}^*\phi \qquad \tilde{\nabla} \times \tilde{\nabla}^*\phi = \tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi + i\cdot\tilde{\nabla}\circ\tilde{\nabla}^*\phi$$

$$\tilde{\nabla}^T \times \tilde{\nabla}\phi = \tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi - i\cdot\tilde{\nabla}\circ\tilde{\nabla}^*\phi \qquad \tilde{\nabla}^A \times \tilde{\nabla}^*\phi = \tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi + i\cdot\tilde{\nabla}\circ\tilde{\nabla}^*\phi$$

$$\tilde{\nabla}^A \times \tilde{\nabla}\phi = i\cdot\tilde{\nabla}^2\phi \qquad \tilde{\nabla}^T \times \tilde{\nabla}^*\phi = -i\cdot\tilde{\nabla}^2\phi$$

Crossed-curl of a Photor Gradient and it's Conjugate:

$$\tilde{\nabla} \otimes \tilde{\nabla}\phi = -i\cdot\tilde{\nabla}\circ\tilde{\nabla}^*\phi \qquad \tilde{\nabla}^* \otimes \tilde{\nabla}^*\phi = i\cdot\tilde{\nabla}\circ\tilde{\nabla}^*\phi$$

$$\tilde{\nabla}^* \otimes \tilde{\nabla}\phi = \tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi + i\cdot\tilde{\nabla}^2\phi \qquad \tilde{\nabla} \otimes \tilde{\nabla}^*\phi = \tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi - i\cdot\tilde{\nabla}^2\phi$$

$$\tilde{\nabla}^T \otimes \tilde{\nabla}\phi = -i\cdot\tilde{\nabla}\circ\tilde{\nabla}^*\phi \qquad \tilde{\nabla}^A \otimes \tilde{\nabla}^*\phi = i\cdot\tilde{\nabla}\circ\tilde{\nabla}^*\phi$$

$$\tilde{\nabla}^A \otimes \tilde{\nabla}\phi = \tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi + i\cdot\tilde{\nabla}^2\phi \qquad \tilde{\nabla}^T \otimes \tilde{\nabla}^*\phi = \tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi - i\cdot\tilde{\nabla}^2\phi$$

To conjugate a curl, conjugate both the Curl-operator, and the Photor-operand, just as in Complex analyse. All the calculations above can be expressed in tensor notation, but the classification would be almost or totally absent. The Faraday tensor ($F_{\alpha\beta}$) is the closest relative to the 8 Photor Curls.

2.2 The Faraday Tensor and its relation to Photors:

The classical Electromagnetic Field Strength Tensor - the Faraday Tensor - is defined by:

$$[F]=\begin{bmatrix} 0 & B_z & -B_y & \frac{-i}{c}\cdot E_x \\ -B_z & 0 & B_x & \frac{-i}{c}\cdot E_y \\ B_y & -B_x & 0 & \frac{-i}{c}\cdot E_z \\ \frac{i}{c}\cdot E_x & \frac{i}{c}\cdot E_y & \frac{i}{c}\cdot E_z & 0 \end{bmatrix}$$

The classical electromagnetic stress energy momentum tensor is constructed from the products:

$$[F]\cdot[F]=\begin{bmatrix} 0 & B_z & -B_y & \frac{-i}{c}\cdot E_x \\ -B_z & 0 & B_x & \frac{-i}{c}\cdot E_y \\ B_y & -B_x & 0 & \frac{-i}{c}\cdot E_z \\ \frac{i}{c}\cdot E_x & \frac{i}{c}\cdot E_y & \frac{i}{c}\cdot E_z & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & B_z & -B_y & \frac{-i}{c}\cdot E_x \\ -B_z & 0 & B_x & \frac{-i}{c}\cdot E_y \\ B_y & -B_x & 0 & \frac{-i}{c}\cdot E_z \\ \frac{i}{c}\cdot E_x & \frac{i}{c}\cdot E_y & \frac{i}{c}\cdot E_z & 0 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1}{c^2}\cdot E_x{}^2-B_z{}^2-B_y{}^2 & \frac{1}{c^2}\cdot E_xE_y+B_xB_y & \frac{1}{c^2}\cdot E_xE_z+B_xB_z & \frac{i}{c}\cdot\left(E_zB_y-E_yB_z\right) \\ \frac{1}{c^2}\cdot E_xE_y+B_xB_y & \frac{1}{c^2}\cdot E_y{}^2-B_x{}^2-B_z{}^2 & \frac{1}{c^2}\cdot E_yE_z+B_yB_z & \frac{i}{c}\cdot\left(E_xB_z-E_zB_x\right) \\ \frac{1}{c^2}\cdot E_xE_z+B_xB_z & \frac{1}{c^2}\cdot E_yE_z+B_yB_z & \frac{1}{c^2}\cdot E_z{}^2-B_x{}^2-B_y{}^2 & \frac{i}{c}\cdot\left(E_xB_y-E_yB_x\right) \\ \frac{i}{c}\cdot\left(E_zB_y-E_yB_z\right) & \frac{i}{c}\cdot\left(E_xB_z-E_zB_x\right) & \frac{i}{c}\cdot\left(E_xB_y-E_yB_x\right) & \frac{1}{c^2}\cdot\vec{E}^2 \end{bmatrix}$$

$$[F]\cdot[F]^C=\begin{bmatrix} 0 & B_z & -B_y & \frac{-i}{c}\cdot E_x \\ -B_z & 0 & B_x & \frac{-i}{c}\cdot E_y \\ B_y & -B_x & 0 & \frac{-i}{c}\cdot E_z \\ \frac{i}{c}\cdot E_x & \frac{i}{c}\cdot E_y & \frac{i}{c}\cdot E_z & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & B_z & -B_y & \frac{i}{c}\cdot E_x \\ -B_z & 0 & B_x & \frac{i}{c}\cdot E_y \\ B_y & -B_x & 0 & \frac{i}{c}\cdot E_z \\ \frac{-i}{c}\cdot E_x & \frac{-i}{c}\cdot E_y & \frac{-i}{c}\cdot E_z & 0 \end{bmatrix}$$

$$=\begin{bmatrix} \frac{-1}{c^2}\cdot E_x{}^2-B_z{}^2-B_y{}^2 & \frac{-1}{c^2}\cdot E_xE_y+B_xB_y & \frac{-1}{c^2}\cdot E_xE_z+B_xB_z & \frac{-i}{c}\cdot\left(E_zB_y-E_yB_z\right) \\ \frac{-1}{c^2}\cdot E_xE_y+B_xB_y & \frac{-1}{c^2}\cdot E_y{}^2-B_x{}^2-B_z{}^2 & \frac{-1}{c^2}\cdot E_yE_z+B_yB_z & \frac{-i}{c}\cdot\left(E_xB_z-E_zB_x\right) \\ \frac{-1}{c^2}\cdot E_xE_z+B_xB_z & \frac{-1}{c^2}\cdot E_yE_z+B_yB_z & \frac{-1}{c^2}\cdot E_z{}^2-B_x{}^2-B_y{}^2 & \frac{-i}{c}\cdot\left(E_xB_y-E_yB_x\right) \\ \frac{i}{c}\cdot\left(E_zB_y-E_yB_z\right) & \frac{i}{c}\cdot\left(E_xB_z-E_zB_x\right) & \frac{i}{c}\cdot\left(E_xB_y-E_yB_x\right) & \frac{-1}{c^2}\cdot\vec{E}^2 \end{bmatrix}$$

The general expression for the diagonal and off-diagonal contribution is…

2.3 Cross-Curls and Crossed-Curls generalize the Faraday Tensor.

We will benefit by defining a new *Electric Potential* (**Φ**=c **A**) mathematically equivalent to the classical vector potential (**A**) which is in fact a momentum per charge as seen by the relation "$\mathbf{p}_\pi$=m**v** + q**A**".

Cross-curls of the ***Electric Potential (Φ)*** generates four ***Faraday-Fields*** of the ***Time Transpose*** kind:

$$\tilde{\nabla} \times \tilde{\Phi} = c \cdot \vec{B} - \vec{E} + i \cdot \tilde{\nabla} \circ \tilde{\Phi} \qquad \tilde{\nabla} \times \tilde{\Phi}^* = c \cdot \vec{B} + \vec{E}^* + i \cdot \tilde{\nabla} \circ \tilde{\Phi}^*$$
$$\tilde{\nabla}^A \times \tilde{\Phi} = -c \cdot \vec{B} - \vec{E} + i \cdot \tilde{\nabla} \circ \tilde{\Phi} \qquad \tilde{\nabla}^A \times \tilde{\Phi}^* = -c \cdot \vec{B} + \vec{E}^* + i \cdot \tilde{\nabla} \circ \tilde{\Phi}^*$$
$$\tilde{\nabla}^C \times \tilde{\Phi} = c \cdot \vec{B} + \vec{E}^* - i \cdot \tilde{\nabla}^* \circ \tilde{\Phi} \qquad \tilde{\nabla}^C \times \tilde{\Phi}^* = c \cdot \vec{B} - \vec{E} - i \cdot \tilde{\nabla} \circ \tilde{\Phi}$$
$$\tilde{\nabla}^T \times \tilde{\Phi} = -c \cdot \vec{B} + \vec{E}^* - i \cdot \tilde{\nabla}^* \circ \tilde{\Phi} \qquad \tilde{\nabla}^T \times \tilde{\Phi}^* = -c \cdot \vec{B} - \vec{E} - i \cdot \tilde{\nabla} \circ \tilde{\Phi}$$

Crossed-curls of ***Electric Potentials***, generate four ***Faraday-Fields*** of the ***Time Adjoint*** kind:

$$\tilde{\nabla} \otimes \tilde{\Phi} = c \cdot \vec{B} - \vec{E} - i \cdot \tilde{\nabla}^* \circ \tilde{\Phi} \qquad \tilde{\nabla} \otimes \tilde{\Phi}^* = c \cdot \vec{B} + \vec{E}^* - i \cdot \tilde{\nabla} \circ \tilde{\Phi}$$
$$\tilde{\nabla}^T \otimes \tilde{\Phi} = -c \cdot \vec{B} - \vec{E} - i \cdot \tilde{\nabla}^* \circ \tilde{\Phi} \qquad \tilde{\nabla}^T \otimes \tilde{\Phi}^* = -c \cdot \vec{B} + \vec{E}^* - i \cdot \tilde{\nabla} \circ \tilde{\Phi}$$
$$\tilde{\nabla}^C \otimes \tilde{\Phi} = c \cdot \vec{B} + \vec{E}^* + i \cdot \tilde{\nabla} \circ \tilde{\Phi} \qquad \tilde{\nabla}^C \otimes \tilde{\Phi}^* = c \cdot \vec{B} - \vec{E} + i \cdot \tilde{\nabla} \circ \tilde{\Phi}^*$$
$$\tilde{\nabla}^A \otimes \tilde{\Phi} = -c \cdot \vec{B} + \vec{E}^* + i \cdot \tilde{\nabla} \circ \tilde{\Phi} \qquad \tilde{\nabla}^A \otimes \tilde{\Phi}^* = -c \cdot \vec{B} - \vec{E} + i \cdot \tilde{\nabla} \circ \tilde{\Phi}^*$$

Cross-curls (or ***Crossed-curls***) transform ***Time Reflection*** into ***Negative Transpose*** (or ***Negative Adjoint***):

$$\tilde{\nabla} \times \underset{(t \to -t)}{=} -\tilde{\nabla}^T \times \qquad \tilde{\nabla} \otimes \underset{(t \to -t)}{=} -\tilde{\nabla}^A \otimes$$
$$\tilde{\nabla}^A \times \underset{(t \to -t)}{=} -\tilde{\nabla}^C \times \qquad \tilde{\nabla}^T \otimes \underset{(t \to -t)}{=} -\tilde{\nabla}^C \otimes$$
$$\tilde{\nabla}^C \times \underset{(t \to -t)}{=} -\tilde{\nabla}^A \times \qquad \tilde{\nabla}^C \otimes \underset{(t \to -t)}{=} -\tilde{\nabla}^T \otimes$$
$$\tilde{\nabla}^T \times \underset{(t \to -t)}{=} -\tilde{\nabla} \times \qquad \tilde{\nabla}^A \otimes \underset{(t \to -t)}{=} -\tilde{\nabla} \otimes$$

Rearrange the ***Cross-curls*** into the order (A,C,T) and rearrange the ***Crossed-curls*** into the order (T,C,A). Each ***Photor Curl*** can now be expressed compactly using ordinal integers (n=0,1,2,3):

$$Cross\,curl_n(\tilde{\Phi}) = \tfrac{1}{c} \cdot \tfrac{\partial}{\partial t}\tilde{\Phi} + (-1)^n \cdot \vec{\nabla} \times \vec{\Phi} + (-1)^{[n/2]} \cdot \left(\vec{\nabla}\phi + i \cdot \vec{\nabla} \circ \vec{\Phi}\right)$$
$$Crossed\,curl_n(\tilde{\Phi}) = \tfrac{1}{c} \cdot \tfrac{\partial}{\partial t}\tilde{\Phi} + (-1)^n \cdot \vec{\nabla} \times \vec{\Phi} + (-1)^{[n/2]} \cdot \left(\vec{\nabla}\phi - i \cdot \vec{\nabla} \circ \vec{\Phi}\right)$$

The square bracket [*n/2*] in the exponent represents the integer part of its argument. This ***Integer Function*** will now be used, to unite ***Cross-curls*** and ***Crossed-curls*** into a ***Photor-curl$_n$*** with (n=0,1,2,3,4,5,6,7):

$$curl_n(\tilde{\Phi}) = \tfrac{1}{c} \cdot \tfrac{\partial}{\partial t}\tilde{\Phi} \; + \; e^{i \cdot \pi \cdot [n/1]} \cdot \vec{\nabla} \times \vec{\Phi} \; + \; e^{i \cdot \pi \cdot [n/2]} \cdot \vec{\nabla}\phi \; + \; i \cdot e^{i \cdot \pi \cdot [n/4]} \cdot \vec{\nabla} \circ \vec{\Phi}$$

We conclude, that the ***Eight Photor Curls*** give a complete permutation of all spatial derivatives. The spatial derivatives are the ***Vector Curl***, the ***Vector Gradient*** and the ***Vector Divergence***.

2.4 The Charge Density - Gauge equations:

In 3D vector geometry the divergence of the curl is zero. Investigate the Cross-Curl and the Crossed-Curl by evaluating the Photor divergence. Simplify in terms of the Gauge (g) and it's conjugate (g^*).

$$\tilde{\nabla} \circ \left(\tilde{\nabla} \times \tilde{\Phi}\right) = \tilde{\nabla} \circ \left(\tilde{\nabla}^A \times \tilde{\Phi}\right) = \tfrac{\partial}{\partial t} g - \rho / \varepsilon$$
$$\tilde{\nabla} \circ \left(\tilde{\nabla}^* \times \tilde{\Phi}\right) = \tilde{\nabla} \circ \left(\tilde{\nabla}^T \times \tilde{\Phi}\right) = \tfrac{\partial}{\partial t} g + \rho / \varepsilon$$
$$\tilde{\nabla}^* \circ \left(\tilde{\nabla} \times \tilde{\Phi}\right) = \tilde{\nabla}^* \circ \left(\tilde{\nabla}^A \times \tilde{\Phi}\right) = -\tfrac{\partial}{\partial t} g - \rho / \varepsilon$$
$$\tilde{\nabla}^* \circ \left(\tilde{\nabla}^* \times \tilde{\Phi}\right) = \tilde{\nabla}^* \circ \left(\tilde{\nabla}^T \times \tilde{\Phi}\right) = \tfrac{\partial}{\partial t} g + 2 \cdot \tfrac{\partial}{\partial t} g^* + \rho / \varepsilon$$

$$\tilde{\nabla}^* \circ \left(\tilde{\nabla} \otimes \tilde{\Phi}\right) = \tilde{\nabla}^* \circ \left(\tilde{\nabla}^T \otimes \tilde{\Phi}\right) = \tfrac{\partial}{\partial t} g^* - \rho / \varepsilon$$
$$\tilde{\nabla}^* \circ \left(\tilde{\nabla}^* \otimes \tilde{\Phi}\right) = \tilde{\nabla}^* \circ \left(\tilde{\nabla}^A \otimes \tilde{\Phi}\right) = \tfrac{\partial}{\partial t} g^* + \rho / \varepsilon$$
$$\tilde{\nabla} \circ \left(\tilde{\nabla} \otimes \tilde{\Phi}\right) = \tilde{\nabla} \circ \left(\tilde{\nabla}^T \otimes \tilde{\Phi}\right) = -\tfrac{\partial}{\partial t} g^* - \rho / \varepsilon$$
$$\tilde{\nabla} \circ \left(\tilde{\nabla}^* \otimes \tilde{\Phi}\right) = \tilde{\nabla} \circ \left(\tilde{\nabla}^A \otimes \tilde{\Phi}\right) = \tfrac{\partial}{\partial t} g^* + 2 \cdot \tfrac{\partial}{\partial t} g + \rho / \varepsilon$$

Observe the appearance of six unique objects registering as a Charge Density and/or a temporal Gauge variation. We are expressing the connection between the Charge Density, and the Gauge:

$$\rho_0 = \varepsilon_0 \cdot \tfrac{\partial}{\partial t} g - G_0 \cdot \tilde{\nabla} \circ \tilde{\nabla} \times \tilde{A} = \varepsilon_0 \cdot \tfrac{\partial}{\partial t} g^* - G_0 \cdot \tilde{\nabla}^* \circ \tilde{\nabla} \otimes \tilde{A}$$

We restore subscripts, recall our previously defined Conductance (G_0), and drop parentheses for clarity.

2.5 The Zero Divergence Curl Equations:

To investigate further into the realms of the curls, let us rewrite "**div curl Φ**" in terms of the scalar potential (ϕ) and the Charge Density (ρ), and then equate each to zero. In order of simplicity we get:

$$\vec{\nabla}^2 \phi - \tfrac{1}{c^2} \cdot \tfrac{\partial^2}{\partial t^2} \phi = 0 \qquad \vec{\nabla}^2 \phi + \tfrac{1}{c^2} \cdot \tfrac{\partial^2}{\partial t^2} \phi = 0$$
$$\vec{\nabla}^2 \phi - \tfrac{1}{c^2} \cdot \tfrac{\partial^2}{\partial t^2} \phi = -2 \cdot \rho / \varepsilon \qquad \vec{\nabla}^2 \phi + \tfrac{1}{c^2} \cdot \tfrac{\partial^2}{\partial t^2} \phi = -2 \cdot \rho / \varepsilon$$
$$\vec{\nabla}^2 \phi - \tfrac{1}{3 \cdot c^2} \cdot \tfrac{\partial^2}{\partial t^2} \phi = -\tfrac{2}{3} \cdot \rho / \varepsilon \qquad \vec{\nabla}^2 \phi + \tfrac{1}{3 \cdot c^2} \cdot \tfrac{\partial^2}{\partial t^2} \phi = -\tfrac{2}{3} \cdot \rho / \varepsilon$$

If we treat the spatial and temporal contribution on equal footing, fractional Charge Density, appear. To get a better view of the situation let us compare the Gauge in each case:

$$\rho = -\varepsilon \cdot \tfrac{\partial}{\partial t} g \qquad \rho = -\varepsilon \cdot \tfrac{\partial}{\partial t} g^*$$
$$\rho = +\varepsilon \cdot \tfrac{\partial}{\partial t} g \qquad \rho = +\varepsilon \cdot \tfrac{\partial}{\partial t} g^*$$
$$\rho = -\varepsilon \cdot \tfrac{\partial}{\partial t} (2 \cdot g + g^*) \qquad \rho = -\varepsilon \cdot \tfrac{\partial}{\partial t} (2 \cdot g^* + g)$$

We are compelled to view charges as a result of an underlying dynamical process!

2.6 The Photor Curl of a Photor Gradient:

The Cross-curl of a Photor Gradient is ***Self-Adjoint*** (or ***Conjugate-Transpose***). The ***Hyperbolic instance*** is a <u>non-dissipative wave equation</u>. The ***Elliptic instance*** is a <u>dissipative particle equation</u>:

$$\tilde{\nabla} \times \tilde{\nabla}\phi = \tilde{\nabla}^{A} \times \tilde{\nabla}\phi = i \cdot \tilde{\nabla}^{2}\phi \qquad \tilde{\nabla} \times \tilde{\nabla}^{*}\phi = \tilde{\nabla}^{A} \times \tilde{\nabla}^{*}\phi = i \cdot \tilde{\nabla}^{*2}\phi + \tfrac{2}{c} \cdot \tfrac{\partial}{\partial t}\vec{\nabla}\phi$$

$$\tilde{\nabla}^{C} \times \tilde{\nabla}^{*}\phi = \tilde{\nabla}^{T} \times \tilde{\nabla}^{*}\phi = -i \cdot \tilde{\nabla}^{2}\phi \qquad \tilde{\nabla}^{C} \times \tilde{\nabla}\phi = \tilde{\nabla}^{T} \times \tilde{\nabla}\phi = -i \cdot \tilde{\nabla}^{*2}\phi + \tfrac{2}{c} \cdot \tfrac{\partial}{\partial t}\vec{\nabla}\phi$$

The Crossed-curl of a Photor Gradient is ***Self-Transpose*** (or ***Conjugate-Adjoint***). The ***Elliptic instance*** is a <u>non-dissipative particle equation</u>. The ***Hyperbolic instance*** is a <u>dissipative wave equation</u>.

$$\tilde{\nabla} \otimes \tilde{\nabla}\phi = \tilde{\nabla}^{T} \otimes \tilde{\nabla}\phi = -i \cdot \tilde{\nabla}^{*2}\phi \qquad \tilde{\nabla} \otimes \tilde{\nabla}^{*}\phi = \tilde{\nabla}^{T} \otimes \tilde{\nabla}^{*}\phi = -i \cdot \tilde{\nabla}^{2}\phi + \tfrac{2}{c} \cdot \tfrac{\partial}{\partial t}\vec{\nabla}\phi$$

$$\tilde{\nabla}^{C} \otimes \tilde{\nabla}^{*}\phi = \tilde{\nabla}^{A} \otimes \tilde{\nabla}^{*}\phi = i \cdot \tilde{\nabla}^{*2}\phi \qquad \tilde{\nabla}^{C} \otimes \tilde{\nabla}\phi = \tilde{\nabla}^{A} \otimes \tilde{\nabla}\phi = i \cdot \tilde{\nabla}^{2}\phi + \tfrac{2}{c} \cdot \tfrac{\partial}{\partial t}\vec{\nabla}\phi$$

The general expression for a Curl of a Gradient can be derived from the General Photor Curl:

$$curl_{n}(\tilde{\nabla}\phi) = i \cdot e^{i \cdot \pi \cdot [n/4]} \cdot \left(\vec{\nabla}^{2}\phi - \frac{e^{-i \cdot \pi \cdot [n/4]}}{c^{2}} \cdot \frac{\partial^{2}}{\partial t^{2}}\phi\right) + \frac{\left(1 - e^{i \cdot \pi \cdot [n/2]}\right)}{c} \cdot \vec{\nabla}\frac{\partial}{\partial t}\phi$$

Observe a term that is either Hyperbolic or Elliptic, and a term that is either Zero or Dissipative.

2.7 The Photor Divergence of a Photor Curl:

The Divergence of the Photor Cross-curl is ***Self-Adjoint*** and ***Conjugate-Transpose***. The ***Hyperbolic instance*** is a <u>non-dissipative wave equation</u>. The ***Elliptic instance*** is a <u>dissipative particle equation</u>:

$$\tilde{\nabla}^{*} \circ \left(\tilde{\nabla} \times \tilde{A}\right) = \tilde{\nabla}^{*} \circ \left(\tilde{\nabla}^{A} \times \tilde{A}\right) = \tfrac{1}{c} \cdot \tilde{\nabla}^{2}\phi \qquad \tilde{\nabla} \circ \left(\tilde{\nabla} \times \tilde{A}\right) = \tilde{\nabla} \circ \left(\tilde{\nabla}^{A} \times \tilde{A}\right) = \tfrac{1}{c} \cdot \tilde{\nabla}^{*2}\phi + \tfrac{2}{c}\tfrac{\partial}{\partial t}\vec{\nabla} \circ \vec{A}$$

$$\tilde{\nabla} \circ \left(\tilde{\nabla}^{C} \times \tilde{A}\right) = \tilde{\nabla} \circ \left(\tilde{\nabla}^{T} \times \tilde{A}\right) = \tfrac{-1}{c} \cdot \tilde{\nabla}^{2}\phi \qquad \tilde{\nabla}^{*} \circ \left(\tilde{\nabla}^{C} \times \tilde{A}\right) = \tilde{\nabla}^{*} \circ \left(\tilde{\nabla}^{T} \times \tilde{A}\right) = \tfrac{-1}{c} \cdot \tilde{\nabla}^{*2}\phi + \tfrac{2}{c}\tfrac{\partial}{\partial t}\vec{\nabla} \circ \vec{A}$$

The Divergence of the Photor Crossed-curl, is ***Self-Transpose*** and ***Conjugate-Adjoint***. The ***Elliptic instance*** is a <u>non-dissipative particle equation</u>. The ***Hyperbolic instance*** is a <u>dissipative wave equation</u>.

$$\tilde{\nabla} \circ \left(\tilde{\nabla} \otimes \tilde{A}\right) = \tilde{\nabla} \circ \left(\tilde{\nabla}^{T} \otimes \tilde{A}\right) = \tfrac{1}{c} \cdot \tilde{\nabla}^{*2}\phi \qquad \tilde{\nabla}^{*} \circ \left(\tilde{\nabla} \otimes \tilde{A}\right) = \tilde{\nabla}^{*} \circ \left(\tilde{\nabla}^{T} \otimes \tilde{A}\right) = \tfrac{1}{c} \cdot \tilde{\nabla}^{2}\phi + \tfrac{2}{c}\tfrac{\partial}{\partial t}\vec{\nabla} \circ \vec{A}$$

$$\tilde{\nabla}^{*} \circ \left(\tilde{\nabla}^{C} \otimes \tilde{A}\right) = \tilde{\nabla}^{*} \circ \left(\tilde{\nabla}^{A} \otimes \tilde{A}\right) = \tfrac{-1}{c} \cdot \tilde{\nabla}^{*2}\phi \qquad \tilde{\nabla} \circ \left(\tilde{\nabla}^{C} \otimes \tilde{A}\right) = \tilde{\nabla} \circ \left(\tilde{\nabla}^{A} \otimes \tilde{A}\right) = \tfrac{-1}{c} \cdot \tilde{\nabla}^{2}\phi + \tfrac{2}{c}\tfrac{\partial}{\partial t}\vec{\nabla} \circ \vec{A}$$

A general expression for a Photor Divergence of a Photor Curl, can be obtained from the Photor $Curl_n$:

$$div\left(curl_{n}(\tilde{A})\right) = \tfrac{1}{c} \cdot e^{i \cdot \pi \cdot [n/2]} \cdot \left(\vec{\nabla}^{2}\phi + \frac{e^{-i \cdot \pi \cdot [n/2]}}{c^{2}} \cdot \frac{\partial^{2}}{\partial t^{2}}\phi\right) + \frac{\left(1 + e^{i \cdot \pi \cdot [n/4]}\right)}{c} \cdot \frac{\partial}{\partial t}\vec{\nabla} \circ \vec{A}$$

Using the fact that "***div (E/c)=*** ρ_0/G_0", the above can be transformed into the "***Fractional Charge Equation***":

$$div\left(curl_{n}(\tilde{A})\right) = \frac{1}{c^{3}} \cdot \frac{\partial^{2}}{\partial t^{2}}\phi - \frac{\left(1 - e^{i \cdot \pi \cdot [n/2]} + e^{i \cdot \pi \cdot [n/4]}\right)}{c} \cdot \vec{\nabla}^{2}\phi - \frac{\left(1 + e^{i \cdot \pi \cdot [n/4]}\right)}{c} \cdot \left(\rho_{0} / \varepsilon_{0}\right)$$

Observe that, when (n=2,3), the coefficient of the Laplace operator is 3, the Charge Density coefficient is 2, while the coefficient of the 2nd order temporal derivative is 1. The name "Fractional" refers to (1/3) and (2/3).

We are now in a peculiar situation: We almost have a Yang-Mills like super-symmetric equation with a Leptonic character and Quark-like colour charge!

2.8 Decomposition into Electric and Magnetic Fields:

Conjugation and transposition allow us to write the Electromagnetic fields in terms of the Photor Potential only. We are free to choose the Cross-curl or the Crossed-curl. Notice the two temporal derivatives:

$$\vec{E} = \tfrac{-1}{2}\cdot\left(\tilde{\nabla}\times - 2\cdot i\cdot\tilde{\nabla}\circ + \tilde{\nabla}^{A}\times\right)\tilde{\Phi} \qquad \vec{B} = \tfrac{1}{2\cdot c}\cdot\left(\tilde{\nabla}\times - \tilde{\nabla}^{A}\times\right)\tilde{\Phi}$$

$$\vec{E}^{*} = \tfrac{1}{2}\cdot\left(\tilde{\nabla}^{*}\times + 2\cdot i\cdot\tilde{\nabla}^{*}\circ + \tilde{\nabla}^{T}\times\right)\tilde{\Phi} \qquad \frac{\partial}{\partial t} = \tfrac{c}{2}\cdot\left(\tilde{\nabla}\times + \tilde{\nabla}^{T}\times\right)$$

$$\vec{E} = \tfrac{-1}{2}\cdot\left(\tilde{\nabla}\otimes + 2\cdot i\cdot\tilde{\nabla}^{*}\circ + \tilde{\nabla}^{T}\otimes\right)\tilde{\Phi} \qquad \vec{B} = \tfrac{1}{2\cdot c}\cdot\left(\tilde{\nabla}\otimes - \tilde{\nabla}^{T}\otimes\right)\tilde{\Phi}$$

$$\vec{E}^{*} = \tfrac{1}{2}\cdot\left(\tilde{\nabla}^{*}\otimes - 2\cdot i\cdot\tilde{\nabla}\circ + \tilde{\nabla}^{A}\otimes\right)\tilde{\Phi} \qquad \frac{\partial}{\partial t} = \tfrac{c}{2}\cdot\left(\tilde{\nabla}\otimes + \tilde{\nabla}^{A}\otimes\right)$$

The two derivatives will allow us to eliminate temporal derivatives in Photor expressions.

2.9 Extending Electric and Magnetic Vectors to Photors

Let us extend the vectors B, D, E, H, M & P from Maxwell's equation to Photors. We do so by introducing respective scalar function b, d, e h, m & p. Use definition for the Cross-curl, and insert objects from Maxwell's equation when appropriate:

$$\tilde{A}^{*}\otimes_{n}\tilde{B} = \tfrac{\phi}{c}\cdot\tilde{B} + e^{i\cdot\pi\cdot[n/1]}\cdot\vec{A}\times\vec{B} + \tfrac{b}{c}\cdot e^{i\cdot\pi\cdot[n/2]}\cdot\vec{A} + i\cdot e^{i\cdot\pi\cdot[n/4]}\cdot\vec{A}\circ\vec{B}$$

We will now show the detailed steps in extending the electromagnetic Vector-Fields into Photor-Fields

$$\begin{aligned}\tilde{\nabla}\times\tilde{E} &= \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{E} + \vec{\nabla}\times\vec{E} + \vec{\nabla}e + i\cdot\left(\vec{\nabla}\circ\vec{E} + \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}e\right)\\ &= \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\left(\vec{E} - c\cdot\vec{B}\right) + \vec{\nabla}e + i\cdot\left(\tfrac{1}{\varepsilon_0}\cdot\rho_T + \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}e\right)\\ &= -\tfrac{\partial}{\partial t}\tilde{\nabla}\times\tilde{A} + \tilde{\nabla}e + i\cdot\left(\tfrac{\partial}{\partial t}g + \tfrac{1}{\varepsilon_0}\cdot\rho_T\right)\\ &= \tilde{\nabla}e - \tfrac{\partial}{\partial t}\left(\tilde{\nabla}\times\tilde{A}\right) - i\cdot c\cdot\tilde{\nabla}^{*}\circ\left(\tilde{\nabla}\times\tilde{A}\right)\\ &= \tilde{\nabla}e - \left(\tfrac{1}{c}\cdot\tfrac{\partial}{\partial t} + i\cdot\tilde{\nabla}^{*}\circ\right)\left(\tilde{\nabla}\times\tilde{\Phi}\right)\end{aligned}$$

$$\begin{aligned}\tilde{\nabla}\times\tilde{B} &= \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{B} + \vec{\nabla}\times\vec{B} + \vec{\nabla}b + i\cdot\tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}b\\ &= \mu\cdot\vec{j}_0 + \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\left(\vec{E} + c\cdot\vec{B}\right) + \vec{\nabla}b + i\cdot\tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}b\\ &= \mu\cdot\tilde{j}_0 - \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\tilde{\nabla}^{A}\times\tilde{A} + \tilde{\nabla}b + i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}g - \tfrac{1}{\varepsilon_0}\cdot\rho_T\right)\end{aligned}$$

$$\begin{aligned}\tilde{\nabla}\times\tilde{D} &= \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{D} + \vec{\nabla}\times\vec{D} + \vec{\nabla}d + i\cdot\left(\vec{\nabla}\circ\vec{D} + \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}d\right)\\ &= \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{P} + \vec{\nabla}\times\vec{P} + \tfrac{\varepsilon_0}{c}\cdot\tfrac{\partial}{\partial t}\left(\vec{E} - c\cdot\vec{B}\right) + \vec{\nabla}d + i\cdot\left(\rho + \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}d\right)\end{aligned}$$

$$\begin{aligned}\tilde{\nabla}\times\tilde{H} &= \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{H} + \vec{\nabla}\times\vec{H} + \vec{\nabla}h + i\cdot\left(\vec{\nabla}\circ\vec{H} + \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}h\right)\\ &= \vec{j}_0 - \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{M} - \vec{\nabla}\times\vec{M} + \varepsilon_0\cdot\tfrac{\partial}{\partial t}\left(\vec{E} + c\cdot\vec{B}\right) + \vec{\nabla}h - i\cdot\left(\vec{\nabla}\circ\vec{M} - \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}h\right)\end{aligned}$$

Notice the Magnetic Scalar Potential being reinvented here in the most natural way.

2.10 The Curl of Curls expanded in vectors and scalars:

There are 16 pure double cross-curls or 32 if we count the set for the conjugated Photor. Starting with the most natural order, fully expanded in vectors and scalars, we have:

$$\tilde{\nabla}\times\tilde{\nabla}\times\tilde{A} = \vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)-\vec{\nabla}^2\vec{A}+\tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\times\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi+\vec{\nabla}g+i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}+\vec{\nabla}^2\phi+\tfrac{\partial}{\partial t}g\right)$$
$$\tilde{\nabla}\times\tilde{\nabla}\times\tilde{A}^* = \vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)-\vec{\nabla}^2\vec{A}+\tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\times\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}-\tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi+\vec{\nabla}g^*+i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}-\vec{\nabla}^2\phi+\tfrac{\partial}{\partial t}g^*\right)$$

$$\tilde{\nabla}\times\tilde{\nabla}^*\times\tilde{A} = -\vec{\nabla}^2\vec{A}+\tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\times\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}+i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}-\vec{\nabla}^2\phi-\tfrac{\partial}{\partial t}g^*\right)$$
$$\tilde{\nabla}\times\tilde{\nabla}^*\times\tilde{A}^* = -\vec{\nabla}^2\vec{A}+\tfrac{2}{c}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\times\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}+i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}+\vec{\nabla}^2\phi-\tfrac{\partial}{\partial t}g\right)$$

$$\tilde{\nabla}\times\tilde{\nabla}^T\times\tilde{A} = -\vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)+\vec{\nabla}^2\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}-\tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi-\vec{\nabla}g^*+i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}-\vec{\nabla}^2\phi-\tfrac{\partial}{\partial t}g^*\right)$$
$$\tilde{\nabla}\times\tilde{\nabla}^T\times\tilde{A}^* = -\vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)+\vec{\nabla}^2\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi-\vec{\nabla}g+i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}+\vec{\nabla}^2\phi-\tfrac{\partial}{\partial t}g\right)$$

$$\tilde{\nabla}\times\tilde{\nabla}^A\times\tilde{A} = -\vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)+\vec{\nabla}^2\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi+\vec{\nabla}g+i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}+\vec{\nabla}^2\phi+\tfrac{\partial}{\partial t}g\right)$$
$$\tilde{\nabla}\times\tilde{\nabla}^A\times\tilde{A}^* = -\vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)+\vec{\nabla}^2\vec{A}+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A}-\tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi+\vec{\nabla}g^*+i\cdot\tfrac{1}{c}\cdot\left(\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}-\vec{\nabla}^2\phi+\tfrac{\partial}{\partial t}g^*\right)$$

Contract the above to Photors, whenever possible, else, use the Electric Field or its conjugate:

$$\tilde{\nabla}\times\tilde{\nabla}\times\tilde{A} = -\tilde{\nabla}^2\tilde{A}^*+\tfrac{2}{c}\cdot\vec{\nabla}\times\vec{E}^*+2\cdot\tilde{\nabla}^*g$$
$$\tilde{\nabla}\times\tilde{\nabla}^*\times\tilde{A} = -\tilde{\nabla}^2\tilde{A}-\tfrac{2}{c}\cdot\vec{\nabla}\times\vec{E}$$
$$\tilde{\nabla}\times\tilde{\nabla}^T\times\tilde{A} = \left(\tilde{\nabla}\circ\tilde{\nabla}^*\right)\tilde{A}^*-\tilde{\nabla}g-\tilde{\nabla}^*g^*$$
$$\tilde{\nabla}\times\tilde{\nabla}^A\times\tilde{A} = \left(\tilde{\nabla}\circ\tilde{\nabla}^*\right)\tilde{A}+\tilde{\nabla}^*g-\tilde{\nabla}g^*$$

$$\tilde{\nabla}\times\tilde{\nabla}\times\tilde{A}^* = -\tilde{\nabla}^2\tilde{A}-\tfrac{2}{c}\cdot\vec{\nabla}\times\vec{E}+2\cdot\tilde{\nabla}^*g^*$$
$$\tilde{\nabla}\times\tilde{\nabla}^*\times\tilde{A}^* = -\tilde{\nabla}^2\tilde{A}^*+\tfrac{2}{c}\cdot\vec{\nabla}\times\vec{E}^*$$
$$\tilde{\nabla}\times\tilde{\nabla}^T\times\tilde{A}^* = \left(\tilde{\nabla}\circ\tilde{\nabla}^*\right)\tilde{A}-\tilde{\nabla}^*g-\tilde{\nabla}g^*$$
$$\tilde{\nabla}\times\tilde{\nabla}^A\times\tilde{A}^* = \left(\tilde{\nabla}\circ\tilde{\nabla}^*\right)\tilde{A}^*-\tilde{\nabla}g-\tilde{\nabla}^*g^*$$

The first two pares can be further reduced, with the Photor Current Density from Maxwell's equations:

$$\tilde{\nabla}\times\tilde{\nabla}\times\tilde{A} = \mu\cdot\tilde{J}^*+\tfrac{2}{c}\cdot\vec{\nabla}\times\vec{E}^*+\tilde{\nabla}g^*$$
$$\tilde{\nabla}\times\tilde{\nabla}^*\times\tilde{A} = \mu\cdot\tilde{J}-\tfrac{2}{c}\cdot\vec{\nabla}\times\vec{E}-\tilde{\nabla}g$$

$$\tilde{\nabla}\times\tilde{\nabla}\times\tilde{A}^* = \mu\cdot\tilde{J}^*+\tfrac{2}{c}\cdot\vec{\nabla}\times\vec{E}^*+\tilde{\nabla}^*g^*$$
$$\tilde{\nabla}\times\tilde{\nabla}^*\times\tilde{A}^* = \mu\cdot\tilde{J}-\tfrac{2}{c}\cdot\vec{\nabla}\times\vec{E}-\tilde{\nabla}g^*$$

2.11 The 64 Curls of Curls:

The general equation for the 64 "***curl$_m$(curl$_n$(A))***" with ordinal integers (m,n) can be expressed in terms of the Faraday Field (F_n) and the exponential phase function ($e^{i\pi}$):

$$curl_m\left(curl_n(\widetilde{A})\right) = curl_m\left(\widetilde{F}_n\right) = \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\widetilde{F}_n + e^{i\cdot\pi\cdot[m/1]}\cdot\vec{\nabla}\times\vec{F}_n + e^{i\cdot\pi\cdot[m/2]}\cdot\vec{\nabla}f_n + i\cdot e^{i\cdot\pi\cdot[m/4]}\cdot\vec{\nabla}\circ\vec{F}_n$$

$$\vec{F}_n = re(\widetilde{F}_n) = \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{A} + e^{i\cdot\pi\cdot[n/1]}\cdot\vec{\nabla}\times\vec{A} + \tfrac{1}{c}\cdot e^{i\cdot\pi\cdot[n/2]}\cdot\vec{\nabla}\phi$$
$$f_n = im(\widetilde{F}_n) = \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi + e^{i\cdot\pi\cdot[n/4]}\cdot\vec{\nabla}\circ\vec{A}$$

The Faraday Photor (***F$_n$***) can be separated into a Vector part and Scalar part for a closer examination:

$$re\left(curl_m\left(\widetilde{F}_n\right)\right) = \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\vec{A} - e^{i\cdot\pi\cdot(n+m)}\cdot\vec{\nabla}^2\vec{A} + \left(e^{i\cdot\pi\cdot(n+m)} + e^{i\cdot\pi\cdot([m/2]+[n/4])}\right)\cdot\vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)$$
$$+\tfrac{1}{c}\cdot\left(e^{i\cdot\pi\cdot n} + e^{i\cdot\pi\cdot m}\right)\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\times\vec{A} + \tfrac{1}{c^2}\cdot\left(e^{i\cdot\pi\cdot[n/2]} + e^{i\cdot\pi\cdot[m/2]}\right)\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi$$

$$im\left(curl_m\left(\widetilde{F}_n\right)\right) = \tfrac{1}{c^3}\cdot\tfrac{\partial^2}{\partial t^2}\phi + \tfrac{1}{c}\cdot e^{i\cdot\pi([n/2]+[m/4])}\cdot\vec{\nabla}^2\phi + \tfrac{1}{c}\cdot\left(e^{i\cdot\pi\cdot[n/4]} + e^{i\cdot\pi\cdot[m/4]}\right)\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{A}$$

For convenience, the general equation for the 64 "***curl$_m$(curl$_n$(Φ))***" with ordinal integers (m,n) is also:

$$curl_m\left(curl_n(\widetilde{\Phi})\right) = curl_m\left(\widetilde{F}_n\right) = \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\widetilde{F}_n + e^{i\cdot\pi\cdot[m/1]}\cdot\vec{\nabla}\times\vec{F}_n + e^{i\cdot\pi\cdot[m/2]}\cdot\vec{\nabla}f_n + i\cdot e^{i\cdot\pi\cdot[m/4]}\cdot\vec{\nabla}\circ\vec{F}_n$$

$$\vec{F}_n = re(\widetilde{F}_n) = \tfrac{1}{c}\cdot\tfrac{\partial}{\partial t}\vec{\Phi} + e^{i\cdot\pi\cdot[n/1]}\cdot\vec{\nabla}\times\vec{\Phi} + e^{i\cdot\pi\cdot[n/2]}\cdot\vec{\nabla}\phi$$
$$f_n = im(\widetilde{F}_n) = \tfrac{1}{c^2}\cdot\tfrac{\partial}{\partial t}\phi + e^{i\cdot\pi\cdot[n/4]}\cdot\vec{\nabla}\circ\vec{\Phi}$$

Split into Vector and Scalar for examination:

$$re\left(curl_m\left(\widetilde{F}_n\right)\right) = \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\widetilde{\Phi} - e^{i\cdot\pi\cdot(n+m)}\cdot\vec{\nabla}^2\vec{\Phi} + \left(e^{i\cdot\pi\cdot(n+m)} + e^{i\cdot\pi\cdot([m/2]+[n/4])}\right)\cdot\vec{\nabla}\left(\vec{\nabla}\circ\vec{\Phi}\right)$$
$$+\tfrac{1}{c}\cdot\left(e^{i\cdot\pi\cdot n} + e^{i\cdot\pi\cdot m}\right)\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\times\vec{\Phi} + \tfrac{1}{c}\cdot\left(e^{i\cdot\pi\cdot[n/2]} + e^{i\cdot\pi\cdot[m/2]}\right)\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\phi$$

$$im\left(curl_m\left(\widetilde{F}_n\right)\right) = \tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\phi + e^{i\cdot\pi([n/2]+[m/4])}\cdot\vec{\nabla}^2\phi + \tfrac{1}{c}\cdot\left(e^{i\cdot\pi\cdot[n/4]} + e^{i\cdot\pi\cdot[m/4]}\right)\cdot\tfrac{\partial}{\partial t}\vec{\nabla}\circ\vec{\Phi}$$

Both the Vector part and Scalar part show a Wave equation or a Particle Equation with or without a 1st time derivative with a damping or amplifying character.