4.1 The Luminal Moments (K_L) and (K_M) and their relation to Spin and Torsion:

The cross product "$\boldsymbol{K} = \boldsymbol{I} \times \nabla \times \boldsymbol{\Phi}$" is measured in "Watts/Meter". Here "$\boldsymbol{I} = G_0\, c\, \mathbf{A} = G_0\, \Phi$". To explore, we need two Photor groups, the Lorentz Moment group ($\boldsymbol{K_L}$) representing the Spin, and the Maxwell Moment group ($\boldsymbol{K_M}$) representing Torsion:

$$\widetilde{K}_L = \iota \cdot \vec{E} + c \cdot \vec{I} \times \vec{B} + i \cdot \vec{I} \circ \vec{E} \qquad \widetilde{K}_M = c \cdot \iota \cdot \vec{B} + \vec{I} \times \vec{E} + i \cdot c \cdot \vec{I} \circ \vec{B}$$

$$\widetilde{K}_{LT} = \iota \cdot \vec{E} - c \cdot \vec{I} \times \vec{B} + i \cdot \vec{I} \circ \vec{E} \qquad \widetilde{K}_{MT} = c \cdot \iota \cdot \vec{B} - \vec{I} \times \vec{E} + i \cdot c \cdot \vec{I} \circ \vec{B}$$

$$\widetilde{K}_{LC} = \iota \cdot \vec{E}^* + c \cdot \vec{I} \times \vec{B} + i \cdot \vec{I} \circ \vec{E}^* \qquad \widetilde{K}_{MC} = c \cdot \iota \cdot \vec{B} + \vec{I} \times \vec{E}^* + i \cdot c \cdot \vec{I} \circ \vec{B}$$

$$\widetilde{K}_{LA} = \iota \cdot \vec{E}^* - c \cdot \vec{I} \times \vec{B} + i \cdot \vec{I} \circ \vec{E}^* \qquad \widetilde{K}_{MA} = c \cdot \iota \cdot \vec{B} - \vec{I} \times \vec{E}^* + i \cdot c \cdot \vec{I} \circ \vec{B}$$

Take care to discriminate between the magnetic scalar potential (iota) and the complex number (i).

$$\tfrac{1}{c} \cdot \widetilde{K}_L = \tfrac{1}{2} \cdot \widetilde{\nabla} \widetilde{I}^2 - \left(\widetilde{I} \circ \widetilde{\nabla}\right) \widetilde{I}$$

$$\tfrac{1}{c} \cdot \widetilde{K}_{LT} = \tfrac{-1}{2} \cdot \widetilde{\nabla}^* \left(\widetilde{I}^* \widetilde{I}\right) + \left(\widetilde{I} \circ \widetilde{\nabla}^*\right) \widetilde{I}^*$$

$$\tfrac{1}{c} \cdot \widetilde{K}_{LC} = \tfrac{1}{2} \cdot \widetilde{\nabla}^* \widetilde{I}^2 - \left(\widetilde{I} \circ \widetilde{\nabla}^*\right) \widetilde{I}$$

$$\tfrac{1}{c} \cdot \widetilde{K}_{LA} = \tfrac{-1}{2} \cdot \widetilde{\nabla} \left(\widetilde{I}^* \widetilde{I}\right) + \left(\widetilde{I} \circ \widetilde{\nabla}\right) \widetilde{I}^*$$

$$\widetilde{K}_M = \left(\vec{\nabla}\phi + \tfrac{\partial}{\partial t}\vec{A}\right) \times \vec{I} + \left(\phi + i \cdot \vec{\Phi} \circ\right)\left(\vec{\nabla} \times \vec{I}\right)$$

$$\widetilde{K}_{MT} = -\left(\vec{\nabla}\phi + \tfrac{\partial}{\partial t}\vec{A}\right) \times \vec{I} + \left(\phi + i \cdot \vec{\Phi} \circ\right)\left(\vec{\nabla} \times \vec{I}\right)$$

$$\widetilde{K}_{MC} = \left(\vec{\nabla}\phi - \tfrac{\partial}{\partial t}\vec{A}\right) \times \vec{I} + \left(\phi + i \cdot \vec{\Phi} \circ\right)\left(\vec{\nabla} \times \vec{I}\right)$$

$$\widetilde{K}_{MA} = -\left(\vec{\nabla}\phi - \tfrac{\partial}{\partial t}\vec{A}\right) \times \vec{I} + \left(\phi + i \cdot \vec{\Phi} \circ\right)\left(\vec{\nabla} \times \vec{I}\right)$$

We use the potentials ($\boldsymbol{A}$), ($\boldsymbol{\Phi}$) or ($\boldsymbol{I}$) at will. To explore the Lorentz Spin and the Maxwell Torsion further, evaluate each of their Photor Divergence, starting with the simpler of the two, the Maxwell Torsion:

$$\widetilde{\nabla} \circ \widetilde{K}_M = -2 \cdot \vec{I} \circ \left(\vec{\nabla} \times \vec{E}\right) \qquad \widetilde{\nabla}^* \circ \widetilde{K}_{MC} = -2 \cdot \vec{I} \circ \left(\vec{\nabla} \times \vec{E}^*\right)$$

$$\widetilde{\nabla} \circ \widetilde{K}_{MT} = -2 \cdot \vec{E} \circ \left(\vec{\nabla} \times \vec{I}\right) \qquad \widetilde{\nabla}^* \circ \widetilde{K}_{MA} = -2 \cdot \vec{E}^* \circ \left(\vec{\nabla} \times \vec{I}\right)$$

$$\widetilde{\nabla} \circ \widetilde{K}_{MC} = 2 \cdot \left(\vec{\nabla} \times \vec{I}\right) \circ \left(\vec{E}^* + \vec{\nabla}\phi\right) \qquad \widetilde{\nabla}^* \circ \widetilde{K}_M = 2 \cdot \left(\vec{\nabla} \times \vec{I}\right) \circ \left(\vec{E} + \vec{\nabla}\phi\right)$$

$$\widetilde{\nabla} \circ \widetilde{K}_{MA} = 2 \cdot \left(\vec{\nabla} \times \vec{I}\right) \circ \vec{\nabla}\phi \qquad \widetilde{\nabla}^* \circ \widetilde{K}_{MT} = 2 \cdot \left(\vec{\nabla} \times \vec{I}\right) \circ \vec{\nabla}\phi$$

Note that ($\boldsymbol{K_M}$) will vanish if ($\boldsymbol{I} = \boldsymbol{A}/\mu$) is orthogonal to ($\partial \boldsymbol{B}/\partial t$). Further, ($\boldsymbol{K_{MT}}$) does the same if ($\boldsymbol{E}$) is orthogonal to ($\boldsymbol{B}$). We conclude, that the Maxwell Moment is "small" in macroscopic engineering electricity, just as the Maxwell Force is "small" when the Current Density ($\boldsymbol{j_0}$), is orthogonal to ($\boldsymbol{B}$) and parallel to ($\boldsymbol{E}$).

$$\tfrac{1}{c} \cdot \widetilde{\nabla} \circ \widetilde{K}_L = -\varepsilon \cdot \vec{E}^2 + \tfrac{1}{\mu} \cdot \vec{B}^2 - \widetilde{A} \circ \widetilde{j}$$

$$\tfrac{1}{c} \cdot \widetilde{\nabla} \circ \widetilde{K}_{LT} = -\varepsilon \cdot \vec{E}^2 - \tfrac{1}{\mu} \cdot \vec{B}^2 + \widetilde{A} \circ \left(\vec{j} - \tfrac{\cdot 2}{c^2} \cdot \tfrac{\partial^2}{\partial t^2} \vec{I} + ic \cdot \rho\right)$$

$$\tfrac{1}{c} \cdot \widetilde{\nabla} \circ \widetilde{K}_{LC} = -\varepsilon \cdot \vec{E} \circ \vec{E}^* + \tfrac{1}{\mu} \cdot \vec{B}^2 - \widetilde{A} \circ \left(\vec{j} - \tfrac{\cdot 2}{c^2} \cdot \tfrac{\partial^2}{\partial t^2} \vec{I} - ic \cdot \rho^*\right)$$

$$\tfrac{1}{c} \cdot \widetilde{\nabla} \circ \widetilde{K}_{LA} = -\varepsilon \cdot \tfrac{1}{2}\left(\vec{E}^2 + \vec{E}^{*2}\right) - \tfrac{1}{\mu} \cdot \vec{B}^2 + \widetilde{A} \circ \left(\vec{j}^* - ic \cdot \rho^*\right)$$

With our choice of potentials, both the Lorentz Spin and the Maxwell Torsion is measured in power per meter [W/m], or force per second [N/s]. Hence, the divergence of it will be energy flux [J/m^{2}s] which is equal to power density [W/m^2]. Power Density divided by speed, is Energy Density [J/m^3], or Pressure [N/m^2].

4.2 The Gauge component of the Luminal Moments (K_G):

The last term in the Luminal Photor force was a "Gauge" term (*g**j***). In the same spirit, we define the last term in the Luminal Moments, both the Lorentz Spin and the Maxwell Torsion, as the Gauge-Moment (*g**I***).

$$\tilde{\nabla}\circ\tilde{K}_G=\tilde{\nabla}\circ\tilde{K}_{GT}=c\cdot\tilde{I}\circ\tilde{\nabla}g+\tfrac{c}{\mu}\cdot g^2 \qquad \tilde{\nabla}^*\circ\tilde{K}_G=\tilde{\nabla}^*\circ\tilde{K}_{GT}=c\cdot\tilde{I}\circ\tilde{\nabla}^*g+\tfrac{c}{\mu}\cdot g\cdot g^*$$

$$\tilde{\nabla}\circ\tilde{K}_{GC}=\tilde{\nabla}\circ\tilde{K}_{GA}=c\cdot\tilde{I}\circ\tilde{\nabla}g^*+\tfrac{c}{\mu}\cdot g\cdot g^* \qquad \tilde{\nabla}^*\circ\tilde{K}_{GC}=\tilde{\nabla}^*\circ\tilde{K}_{GA}=c\cdot\tilde{I}\circ\tilde{\nabla}^*g^*+\tfrac{c}{\mu}\cdot\left(g^*\right)^2$$

As for the Photor force, there are 64 Photor-moments; each composed of three terms. There is always a Lorentz Spin term, a Maxwell Torsion term and a Gauge term:

$$\tilde{I}^T\times\tilde{\nabla}\times\tilde{\Phi}=-\tilde{K}_L+\tilde{K}_M+c\cdot g\cdot\tilde{I} \qquad \tilde{I}^T\otimes\tilde{\nabla}\otimes\tilde{\Phi}=-\tilde{K}_L+\tilde{K}_M+c\cdot g^*\cdot\tilde{I}^*$$

4.3 Photor Divergence of the Luminal Moments (K=IxE):

Many interesting objects appear with the application of the divergence to the moment (***K***):

$$\vec{\nabla}\circ\left(\phi\cdot\vec{E}\right)=-\left(\vec{\nabla}\phi\right)^2-\nabla^2\left(\tfrac{1}{2}\phi^2\right)-\left(\vec{\nabla}\phi\right)\circ\left(\tfrac{\partial}{\partial t}\vec{A}\right)-\phi\cdot\left(\vec{\nabla}\circ\tfrac{\partial}{\partial t}\vec{A}\right)$$

$$\vec{\nabla}\circ\left(\phi\cdot\vec{E}^*\right)=-\left(\vec{\nabla}\phi\right)^2-\nabla^2\left(\tfrac{1}{2}\phi^2\right)+\left(\vec{\nabla}\phi\right)\circ\left(\tfrac{\partial}{\partial t}\vec{A}\right)+\phi\cdot\left(\vec{\nabla}\circ\tfrac{\partial}{\partial t}\vec{A}\right)$$

$$\vec{\nabla}\circ\left(\vec{A}\times\vec{B}\right)=\left(\vec{\nabla}\times\vec{A}\right)^2+\nabla^2\left(\tfrac{1}{2}A^2\right)-\vec{A}\circ\left(\vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)\right)$$

$$\tfrac{\partial}{\partial t}\left(\vec{A}\circ\vec{E}\right)=-\left(\tfrac{\partial}{\partial t}\vec{A}\right)^2-\tfrac{\partial^2}{\partial t^2}\left(\tfrac{1}{2}A^2\right)-\left(\vec{\nabla}\phi\right)\circ\left(\tfrac{\partial}{\partial t}\vec{A}\right)-\vec{A}\circ\left(\vec{\nabla}\tfrac{\partial}{\partial t}\phi\right)$$

$$\tfrac{\partial}{\partial t}\left(\vec{A}\circ\vec{E}^*\right)=\left(\tfrac{\partial}{\partial t}\vec{A}\right)^2+\tfrac{\partial^2}{\partial t^2}\left(\tfrac{1}{2}A^2\right)-\left(\vec{\nabla}\phi\right)\circ\left(\tfrac{\partial}{\partial t}\vec{A}\right)-\vec{A}\circ\left(\vec{\nabla}\tfrac{\partial}{\partial t}\phi\right)$$

$$\vec{\nabla}\circ\left(\phi\cdot\vec{B}\right)=\left(\vec{\nabla}\phi\right)\circ\left(\vec{\nabla}\times\vec{A}\right)$$

$$\vec{\nabla}\circ\left(\vec{A}\times\vec{E}\right)=-\left(\vec{\nabla}\times\vec{A}\right)\circ\left(\tfrac{\partial}{\partial t}\vec{A}\right)-\left(\vec{\nabla}\phi\right)\circ\left(\vec{\nabla}\times\vec{A}\right)+A\circ\left(\vec{\nabla}\times\tfrac{\partial}{\partial t}\vec{A}\right)$$

$$\vec{\nabla}\circ\left(\vec{A}\times\vec{E}^*\right)=+\left(\vec{\nabla}\times\vec{A}\right)\circ\left(\tfrac{\partial}{\partial t}\vec{A}\right)-\left(\vec{\nabla}\phi\right)\circ\left(\vec{\nabla}\times\vec{A}\right)-\vec{A}\circ\left(\vec{\nabla}\times\tfrac{\partial}{\partial t}\vec{A}\right)$$

$$\tfrac{\partial}{\partial t}\left(\vec{A}\circ\vec{B}\right)=\left(\vec{\nabla}\times\vec{A}\right)\circ\left(\tfrac{\partial}{\partial t}\vec{A}\right)+\vec{A}\circ\left(\vec{\nabla}\times\tfrac{\partial}{\partial t}\vec{A}\right)$$

$$\vec{\nabla}\circ\left(g\cdot\vec{A}\right)=\left(\vec{\nabla}\circ\vec{A}\right)^2+\vec{A}\circ\left(\vec{\nabla}\left(\vec{\nabla}\circ\vec{A}\right)\right)+\tfrac{1}{c^2}\cdot\left(\vec{\nabla}\circ\vec{A}\right)\cdot\left(\tfrac{\partial}{\partial t}\phi\right)+\tfrac{1}{c^2}\cdot\vec{A}\circ\left(\vec{\nabla}\tfrac{\partial}{\partial t}\phi\right)$$

$$\tfrac{\partial}{\partial t}\left(g\cdot\phi\right)=\tfrac{1}{c^2}\cdot\left(\tfrac{\partial}{\partial t}\phi\right)^2+\tfrac{1}{c^2}\cdot\tfrac{\partial^2}{\partial t^2}\left(\tfrac{1}{2}\phi^2\right)+\left(\vec{\nabla}\circ\vec{A}\right)\cdot\left(\tfrac{\partial}{\partial t}\phi\right)+\phi\cdot\left(\vec{\nabla}\circ\tfrac{\partial}{\partial t}\vec{A}\right)$$

4.4 Spin, Torsion, Chirality and Helicity:

Recently [23], the Topological Spin Current is given by "$\mathbf{S}_L$ = ϕ**D** + **A**×**H** + i c **A**∘**D**". Spin (ϕ**D** + **A**×**H**), Torsion (ϕ**H** + **I**×**E**), Force (ρ**E** + **j**×**B**) and Moment (cρ**B** + **j**×**E**/c) have similar structure, just permutate the fields. The (**A**∘**E**) term is the Chirality and (**A**∘**B**) is the Helicity:

$$\tilde{S}_L=\phi\cdot\vec{D}+\vec{A}\times\vec{H}+i\cdot c\cdot\vec{A}\circ\vec{D} \qquad \tilde{S}_M=\tfrac{1}{c}\cdot\phi\cdot\vec{H}+c\cdot\vec{A}\times\vec{D}+i\cdot\vec{A}\circ\vec{H}$$

$$\tilde{S}_{LT}=\phi\cdot\vec{D}-\vec{A}\times\vec{H}+i\cdot c\cdot\vec{A}\circ\vec{D} \qquad \tilde{S}_{MT}=\tfrac{1}{c}\cdot\phi\cdot\vec{H}-c\cdot\vec{A}\times\vec{D}+i\cdot\vec{A}\circ\vec{H}$$

$$\tilde{S}_{LC}=\phi\cdot\vec{D}^*+\vec{A}\times\vec{H}+i\cdot c\cdot\vec{A}\circ\vec{D}^* \qquad \tilde{S}_{MC}=\tfrac{1}{c}\cdot\phi\cdot\vec{H}+c\cdot\vec{A}\times\vec{D}^*+i\cdot\vec{A}\circ\vec{H}$$

$$\tilde{S}_{LA}=\phi\cdot\vec{D}^*-\vec{A}\times\vec{H}+i\cdot c\cdot\vec{A}\circ\vec{D}^* \qquad \tilde{S}_{MA}=\tfrac{1}{c}\cdot\phi\cdot\vec{H}-c\cdot\vec{A}\times\vec{D}^*+i\cdot\vec{A}\circ\vec{H}$$